

# SYSTEM COST PROBABILITY DISTRIBUTIONS for ACES ARCHITECTURES

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May 30, 1989

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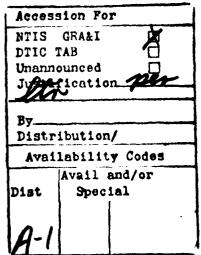
# **TABLE OF CONTENTS**

I	INTRODU	JCTION	1
	PROBLE	M DEFINTION	1
	INPUT I	DATA	2
	CONTEN	NTS	5
П	SUMMAR	RY	6
	RESULT	S FOR INDEPENDENT DISTRIBUTIONS	6
	RESULT	S FOR ALTERNATIVE RISK DISTRIBUTIONS	7
	RESULT:	S FOR DEPENDENT DISTRIBUTIONS	9
Ш		CH USING LOG-NORMAL DISTRIBUTIONS	
		'S	
IV		CH USING NORMAL DISTRIBUTIONS	
V		TITIES TO ASSUMPTIONS	31
	DIFFER	ENTIATING RISK ACCORDING TO PROGRAM	
	PHASE		31
	COMPU	TING COST ESTIMATES ASSUMING PROGRAM	
	INTER- I	DEPENDENCIES	32
APF	PENDIX A:	LOG-NORMAL DISTRIBUTIONS	
	PENDIX B:	CURVE FITS TO INPUT DATA	
API	PENDIX C:	RESULTS FOR LOG-NORMAL DISTRIBUTION FITS	FOR
		DAB ARCHITECTURE ASSUMING INPUT DATA ARE	50%
		CONFIDENCE VALUES	
APF	PENDIX D:	RESULTS FOR LOG-NORMAL DISTRIBUTION FITS	
		DAB ARCHITECTURE ASSUMING INPUT DATA	ARE
A DE	DENIDIVE.	EXPECTED VALUES	DAD
API	PENDIX E:	RESULTS FOR NORMAL DISTRIBUTION FITS FOR ARCHITECTURE ASSUMING INPUT DATA	
		EXPECTED VALUES	AKE
APF	PENDIX F:	SENSITIVITY TO RISK SPLIT BETWEEN PHASES FOR	DAR
1		ARCHITECTURE FOR NORMAL DISTRIBUTION FITS	
APF	PENDIX G:	RESULTS FOR CORRELATED NORMAL DISTRIBUT	ΓΙΟΝ
		FITS FOR ALL ARCHITECTURES	

# **LIST OF FIGURES**

Figure 1	Spring 88 Estimate Updates BSTS Risk Analysis	3
Figure 2	Spring 88 Estimate Updates SSTS Risk Analysis	4
Figure 3	Spring 88 Estimate Updates SBI Risk Analysis	5
Figure 4	Comparison of Upper Bounds of Acquisition Costs	12
Figure 5	Comparison of Acquisition Cost Ranges	13
Figure 6	Comparison of Upper Bounds of Program Costs	14
Figure 7	Comparison of Program Cost Ranges	15
Figure 8	Comparison of Fitted Log Normal Distribution with Input Curve for BSTS	18
Figure 9	Comparison of Fitted Log Normal Distribution with Input Curve for SSTS	19
Figure 10	Comparison of Fitted Log Normal Distribution with Input Curve for SBI	20
Figure 11	Comparison of Fitted Normal Distribution with Input Curve for BSTS	27
Figure 12	Comparison of Fitted Normal Distribution with Input Curve for SSTS	28
Figure 13	Comparison of Fitted Normal Distribution with Input Curve for SBI	29
Figure 14	Program Costs for DAB Architecture	34
Figure 15	Acquisition Costs for DAB Architecture	36
Figure 16	Program Costs for DAB Architecture	37





# LIST OF TABLES

Table 1	Summary of Acquisition and Program Costs Derived with Independent Log-Normal Distributions Assuming Nominal Inputs are 50% Confidence Values of Cost	8
Table 2	Summary of Acquisition and Program Costs Derived with Independent Log-Normal Distributions Assuming Nominal Inputs are Expected Values of Cost	8
Table 3	Summary of Acquisition and Program Costs Derived with Independent Normal Distributions	9
Table 4	Summary of Acquisition and Program Costs Derived with Normal Distributions and Assuming Program Phases and Elements are Correlated with Coefficient 0.50	10
Table 5	Summary of Acquisition and Program Costs when Program Elements are Completly Correlated	10
Table 6	Log-Normal Distribution Definitions	16
Table 7	Log-Normal Fits to Input Data	17
Table 8	Risk Categories Assumed	21
Table 9	Summary of Acquisition and Program Costs Derived with Independent Log-Normal Distributions Assuming Inputs are 50% Confidence Values of Cost	23
Table 10	Summary of Acquisition and Program Costs Derived with Independent Log-Normal Distributions Assuming Inputs are Expected Values of Cost	24
Table 11	Normal Distribution Definitions	25
Table 12	Normal Fits to Input Data	26
Table 13	Summary of Acquisition and Program Costs Derived with Independent Normal Distributions	30
Table 14	Comparison of Architecture Cost Ranges for Two Different Assumptions About the Split Between Program Phase Cost Variations	32
	7 WE SEE TATO	52

# LIST OF ACRONYMS

ACES	Architecture Comparative Evaluation Study
BP	Brilliant Pebbles
BSTS	Boost Surveillance and Tracking System
CCSOIF	Command, Control and System Operating and Integrating
	Functions
DAB	Defense Acquisition Board
<b>ERIS</b>	Exoatmospheric Reentry Vehicle Intercept System
GSTS	Ground-Based Space Surveillance and Tracking System
HEDI	High Endoatmospheric Intercept System
LCH	Launch
MGBR	Midcourse Ground-Based Radar
SBI	Space Based Interceptor
SEI	System Engineering and Integration
SSTS	Space Surveillance and Tracking System
TGBR	Terminal Ground-Based Radar

#### I INTRODUCTION

The purposes of this paper are to present estimates of the probability distributions for the acquisition and program costs for the alternative architectures discussed in the Architecture Comparative Evaluation Study (ACES) [Ref 1], and to describe the procedures we used to develop them. In our first pass through these evaluations, we will assume that the distributions for the cost of R&D and Investment for an element have equal risk and that all cost distributions are statistically independent. Later, we will relax these assumptions.

#### PROBLEM DEFINTION

We are seeking a method for determining the probability distributions of two kinds of system costs: acquisition costs and program costs. The acquisition costs are defined as the costs of the R&D and Investment phases on all the defense elements that are deployed with an architecture. The program costs consists of the R&D costs on all the elements that are to be deployed in the initial deployment, the Investment costs for all the elements deployed in the initial deployment, and the R&D costs of the elements to be deployed in the next phase of deployment. Thus the program costs include all of the acquisitions costs for an initial deployment plus R&D on near term follow-on elements not included initially.

The architectures are ballistic missile defense systems composed of the following sensor, interceptor and battle management elements:

boost surveillance and tracking system (BSTS), space surveillance and tracking system (SSTS), space based interceptor (SBI) or Brilliant Pebbles (BP), ground-based space surveillance and tracking system (GSTS), midcourse ground-based radar (MGBR), exoatmospheric reentry vehicle intercept system (ERIS), terminal ground-based radar (TGBR), high endoatmospheric intercept system (HEDI),

Dyer, JL. et al. "Architecture Comparative Evaluation Study (U)" Volume II, SPARTA Technical Report, McLean, VA <u>SECRET</u> (March 3, 1989)

command, control and system operating and integrating functions (CCSOIF),

system engineering and integration (SEI), and launch (LCH).

The quantities and qualities of each of these elements, and the costquantity estimating relationships are described in the ACES final report.

As we will use the term, the system costs for an architecture includes only the costs for the full scale engineering development (R&D) and deployment (Investment) phase; operations and support (O&S) costs are not included.

#### **INPUT DATA**

The inputs to these evaluations of system cost were 1) the nominal costs for each of two phases (R&D and Investment) for each of the elements to be considered in the near term architectures (from ACES) and 2) the cost distributions from the USAF for the combined costs (R&D plus Investment) for three elements: BSTS, SSTS and SBI. We will use the ACES inputs as the nominal costs (either expected values or 50% confidence values), and the USAF data as the source of the cost risk distributions about those nominal values.

In ACES, we have nominal cost estimates for the appropriate quantities of each element in each of its two phases. We do not have cost risk assessments for all of the elements contained in each of these architectures. The three element cost risk estimates that we do have are presented in Figures 1 through 3 for the BSTS, SSTS and SBI respectively. The nominal values in the figures are assumed to have been superseded by the ACES data. However, we can and will use the nature of the underlying distributions for estimating the cost risk. We will determine the characteristics of the distributions from curve fits to the data we take off of these three figures.

We will develop total system costs (total R&D and Investment) from these input data for three different sets of premises: 1) the element costs are log-normally distributed and the nominal (ACES) costs are estimates at the 50% confidence level, 2) the element costs are log-normally distributed and

the nominal costs are expected value estimates, and 3) the costs are normally distributed and the nominal cost are most likely cost estimates.

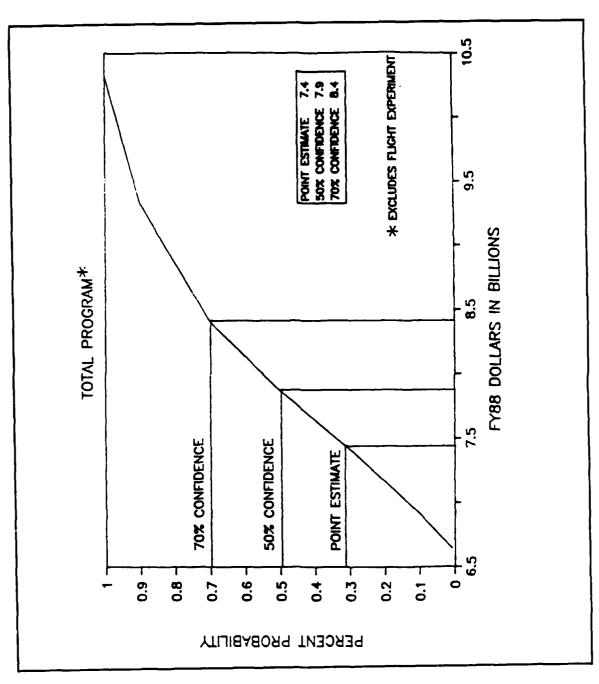


Figure 1. Spring 88 Estimate Updates BSTS Risk Analysis

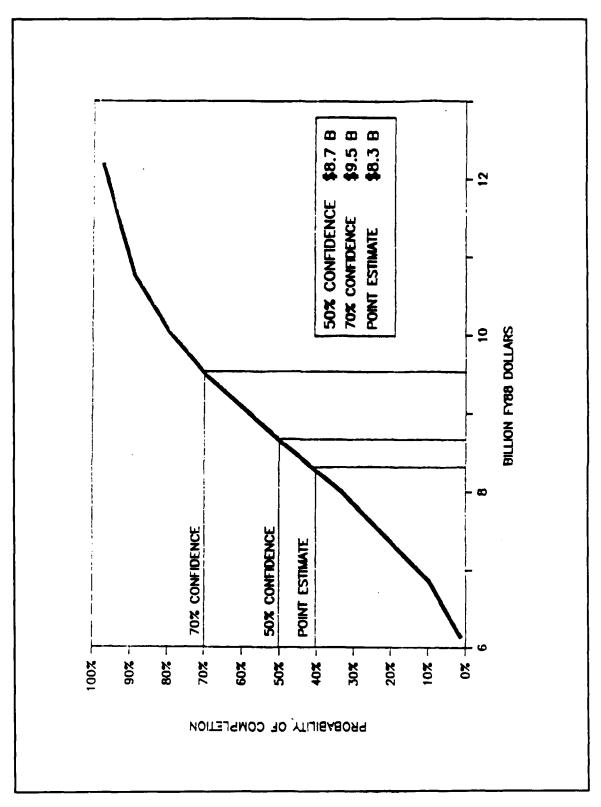


Figure 2. Spring 88 Estimate Updates SSTS Risk Analysis Results

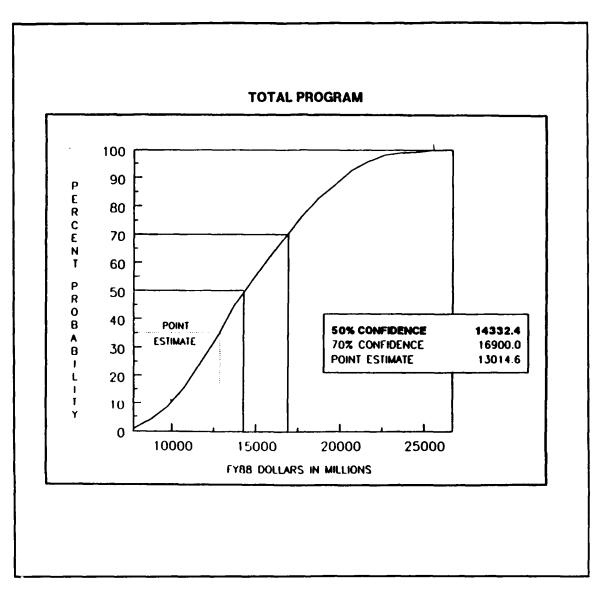


Figure 3. Spring 88 Estimate Updates SBI Risk Analysis Results

#### **CONTENTS**

A summary of the results and our recommended estimates are in the next section. Section II covers the detailed results for the log-normal distribution assumption sets. Section III covers the detailed results for the normal distribution assumption set. In the last section, Section IV, we will examine the sensitivity to our assumptions on risk associated with different program phases and on element independence. A summary of the pertinent features of the log-normal and the normal distribution is presented in Appendix A.

#### II SUMMARY

In this section, we present three sets of results. In the first subsection, we present the system acquisition and program costs for all five architectures assuming that the cost distributions for each element are independent and that the underlying distributions in each phase for each element are not only independent, but have the same distribution. In the second subsection, we examine a change in the nature of the distribution of risks between R&D and Investment phases. In the third subsection, we present the results for the case where distributions for both phases and elements are correlated. For the costs of the architectures, we recommend using the program costs listed in Table 5. Though these costs are derived without rigorous estimation of the correlation between programs, assuming a complete correlation does model the feature that all elements of an architecture operating at the required performance levels in the necessary quantities are needed to make an architecture meet its mission.

#### RESULTS FOR INDEPENDENT DISTRIBUTIONS

In Tables 1 through 3, we present the system acquisition and program costs we have developed for each architecture from each of our sets of premises on the nature of the distributions. As we should expect, all three system costs distributions are relatively tight (low variances) and are relatively close in value to one another. The tightness should be expected as a result of the assumption of independence of the distributions of costs for each phase and for each element. The closeness should be expected because the fitted underlying distributions are approximately equal. (We note that the nominal program reserve - an added cost - is not included here.)

Table 1 presents the estimates we made assuming that: 1) the cost distributions for each phase for each element have <u>log-normal</u> distributions, 2) that both the R&D and Investment phases have equivalent cost risk distributions (equal coefficients of variation), and 3) that the input cost data (nominal values) for each phase for each element were equivalent to cost estimates made at the <u>50% confidence level</u>. The first assumption distinguishes the results of Table 1 from Table 3. The last assumption distinguishes the results of Table 1 from those of Table 2.

Table 2 presents the estimates we made assuming that: 1) the cost distributions for each phase for each element have <u>log-normal</u> distributions, 2) that both the R&D and Investment phases have equivalent cost risk distributions, and 3) that the input cost data (nominal values) for each phase for each element were equivalent to <u>expected value</u> cost estimates.

Table 3 presents the estimates we made assuming that: 1) the cost distributions for each phase for each element have <u>normal</u> distributions, 2) that both the R&D and Investment phases have equivalent cost risk distributions, and 3) that the input cost data for each phase for each element were equivalent to <u>expected value</u> cost estimates (equivalent to 50% confidence estimates for normal distributions).

By the assumption that the input cost data are 50% confidence values the results in Table 1 are biased away from the sum of the nominal costs that is generally cited as the cost of the system. The bias also makes both the lower and upper bounds higher than we see in either Tables 2 or 3. The results of Tables 2 and 3 are essentially equivalent to one another. Except for roundoff differences, the two distributions are everywhere the same.

#### RESULTS FOR ALTERNATIVE RISK DISTRIBUTIONS

The tightness of the distributions seems to belie the intuitive notion that cost estimates should have large ranges when the systems are only now reaching maturity and are still demanding advances in technology. We considered two alternative assumptions: one on the split of the risk between R&D and Investment and the other on independence. Changing the split in risks did not change the essential nature of the distributions. Following the relative sizes of the TRACE accounts in US Army cost estimates, we assumed that R&D has three times the relative risk as the Investment phase. We modeled this by having the coefficient of variation of the R&D be three times the coefficient of variation for the Investment. For the DAB Architecture, it increased the estimated costs by 1B\$.

Table 1
Summary of Acquisition and Program Costs Derived
with Independent Log-Normal Distributions
Assuming Nominal Inputs are 50% Confidence Values of Cost

		Acquisiti	on Costs		
robability	У				
		Archite	ctures		
	ERIS	ERIS/HEDI	DAB	SBICV	BP
0.20	45	62	66	65	37
0.50	47	64	<i>7</i> 0	72	39
0.80	49	66	74	<i>7</i> 9	40
		Progran	n Costs		
robability	У				
		Archite	ectures		
	ERIS	ERIS/HEDI	DAB	SBICV	BP
0.20	55	69	69	<i>7</i> 5	46
0.50	58	71	73	81	48
0.80	60	74	<i>7</i> 7	88	50

# Table 2 Summary of Acquisition and Program Costs Derived with Independent Log-Normal Distributions Assuming Nominal Inputs are Expected Values of Cost

Assuming Nominal Inputs are Expected Values of Cost					
Acquisition Costs					
у					
	Arc	hitectures		<u> </u>	
ERIS	ERIS/HEDI	DAB	SBICV	BP	
44	61	64	64	36	
46	63	68	<b>7</b> 0	38	
48	65	72	77	40	_
	Prog	ram Costs		_	
y					
	Arc	hitectures			
<b>ERIS</b>	ERIS/HEDI	DAB	SBICV	BP	
54	68	67	<b>7</b> 3	46	
57	70	71	<b>7</b> 9	47	
59	73	<b>7</b> 5	86	49	
	ERIS 44 46 48 ERIS 54 57	Acquis  Y  Arc  ERIS ERIS/HEDI  44 61  46 63  48 65  Prog  Y  Arc  ERIS ERIS/HEDI  54 68  57 70	Acquisition Costs  Architectures  ERIS ERIS/HEDI DAB  44 61 64  46 63 68  48 65 72  Program Costs  Architectures  ERIS ERIS/HEDI DAB  54 68 67  57 70 71	Acquisition Costs  Architectures  ERIS ERIS/HEDI DAB SBICV  44 61 64 64  46 63 68 70  48 65 72 77  Program Costs  Architectures  ERIS ERIS/HEDI DAB SBICV  54 68 67 73  57 70 71 79	Acquisition Costs  Architectures  ERIS ERIS/HEDI DAB SBICV BP  44 61 64 64 36  46 63 68 70 38  48 65 72 77 40  Program Costs  Y  Architectures  ERIS ERIS/HEDI DAB SBICV BP  54 68 67 73 46  57 70 71 79 47

Table 3
Summary of Acquisition and Program Costs Derived with Independent Normal Distributions

	W.	un maepenaent N	ormal Dist	ributions	
		Acquis	sition Costs	3	
Probabilit	у				
		Arch	itectures		
	ERIS	ERIS/HEDI	DAB	SBICV	BP
0.20	44	61	64	64	36
0.50	46	63	68	70	38
0.80	48	65	72	77	40
		Prograi	m Costs		
Probabilit	у _				
		Archite	ectures		
	ERIS	ERIS/HEDI	DAB	SBICV	BP
0.20	54	68	68	73	46
0.50	57	70	72	80	48
0.80	59	<i>7</i> 3	<i>7</i> 5	86	50

#### RESULTS FOR DEPENDENT DISTRIBUTIONS

We considered two kinds of interdependence. The first was a dependence between the costs for the R&D phase and for the Investment phase; the second was a dependence between the costs for the different element programs. Of the two, the latter was the more important. We did not have a basis for estimating the covariance between phases or between elements. In order to proceed, we assumed two values of the correlation coefficient, one for between phases and one for between program elements, and we assumed that these correlation coefficients were everywhere the same. We then examined the system costs as a function of changing values in those two coefficients. Thus, the results can only be considered indicative. Of course, the upper bound on the costs cannot be higher than when we assume all program elements are completely correlated. underlying normal distributions, Table 4 presents the system cost results for the assumption that the correlation coefficient both between program phases and between program elements was 0.50. As expected, these costs are higher than those presented in Table 3, but only by about 5%.

Table 4
Summary of Acquisition and Program Costs Derived
with Normal Distributions and Assuming Program Phases and
Elements are Correlated with Coefficient 0.50

			<del></del>	CITTELCTIT GIGG	
		Acquisi	tion Costs		
Probabilit	у				
		Archi	tectures		
	ERIS	ERIS/HEDI	DAB	SBICV	BP
0.20	42	58	61	61	34
0.50	46	63	68	<b>7</b> 0	38
0.80	51	68	75	80	42
		Progra	m Costs		
Probabilit	y		<del> </del>		
		Archit	ectures		
	ERIS	ERIS/HEDI	DAB	SBICV	BP
0.20	51	64	64	70	43
0.50	57	<i>7</i> 0	72	80	48
0.80	62	<i>7</i> 7	<i>7</i> 9	90	52

In Table 5, we present the same data for the case where the programs are completely correlated (the coefficient of correlation between phases is still at 0.50). After adding in the program reserve amount, these are the costs we recommend using for estimates of the costs of the ACES Architectures.

The upper bounds (80% Confidence Values) on the system acquisition costs as a function of the assumed correlation coefficient between elements are compared for each of the five architectures in Figure 4. While the correlation coefficient is not known, its distribution of values will undoubtedly lie somewhere between the bounds given. In the bar graph of Figure 5, we present the system acquisition costs so that both upper (80%) and lower (20%) confidence bounds for correlated and uncorrelated cases and the expected value can be seen. The larger and the smaller values for the upper and lower confidence bounds are for the completely correlated case; the smaller and larger values for the upper and lower confidence bounds are for the completely independent case. The same kinds of data are presented in Figures 6 and 7 for the system program costs. The correlated cost uncertainties for the DAB Architecture are approximately 14%.

Table 5
Summary of Acquisition and Program Costs
when Program Elements are Completly Correlated

	WILCIL	Togram Lientent	s are Com	pietry Correlat	eu	
		Acquis	ition Costs			
Probability	<i>y</i>					
		Arch	itectures			
	<b>ERIS</b>	ERIS/HEDI	DAB	SBICV	BP	
0.20	41	57	59	59	33	
0.50	46	63	68	<b>7</b> 0	38	
0.80	52	69	77	82	43	
		Progr	am Costs			
Probability	7					
		Arch	uitectures			
	<b>ERIS</b>	ERIS/HEDI	DAB	SBICV	BP	
0.20	49	62	62	67	<b>4</b> 1	
0.50	<i>57</i>	70	<i>7</i> 2	80	48	
0.80	64	<i>7</i> 9	82	92	54	

We note in passing that evaluating the completely correlated case with the log-normal distributions instead of with the normal distributions will yield very similar results

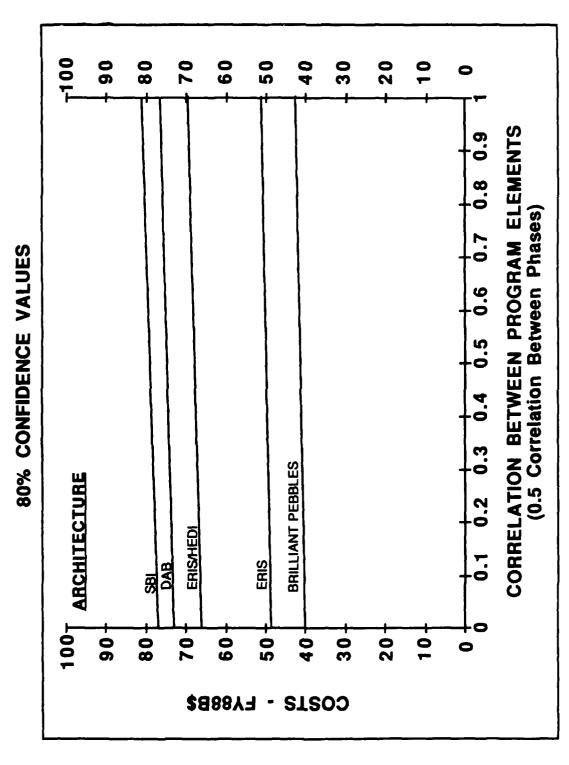


Figure 4. Comparison of Upper Bounds of Acquisition Costs

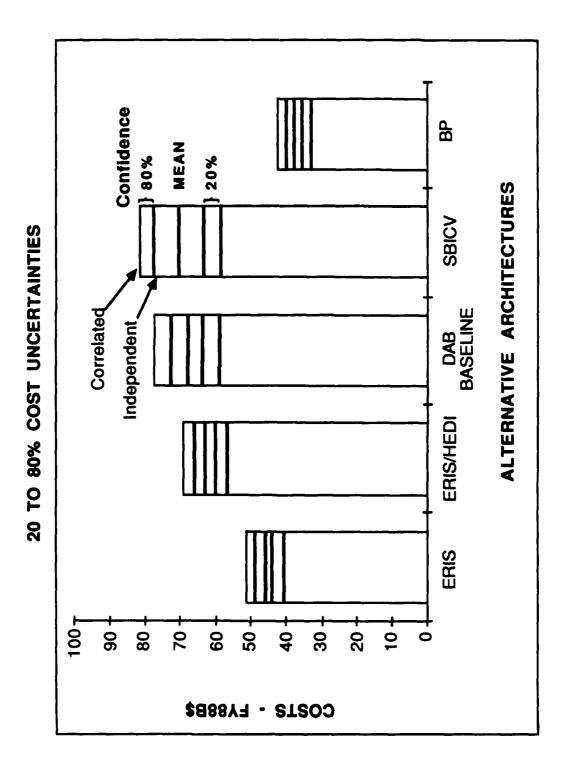


Figure 5. Comparison of Acquisition Cost Ranges

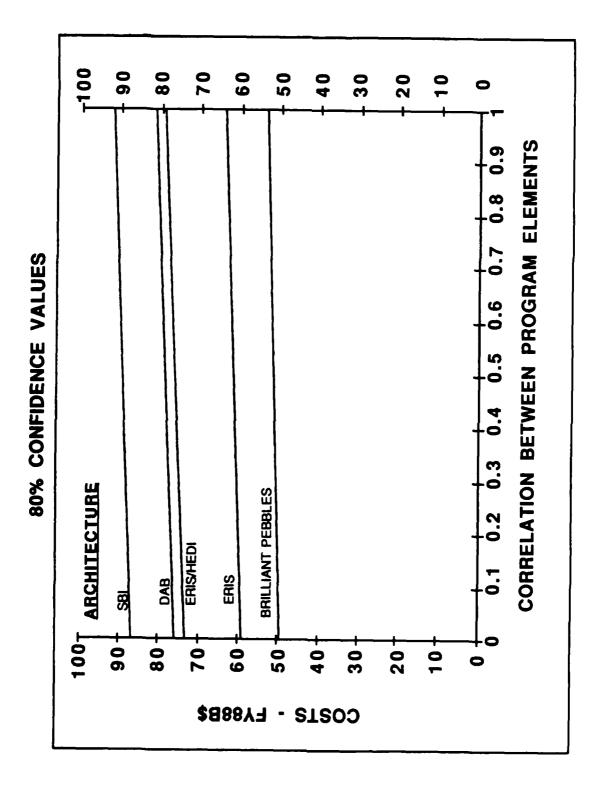


Figure 6. Comparison of Upper Bounds of Program Costs

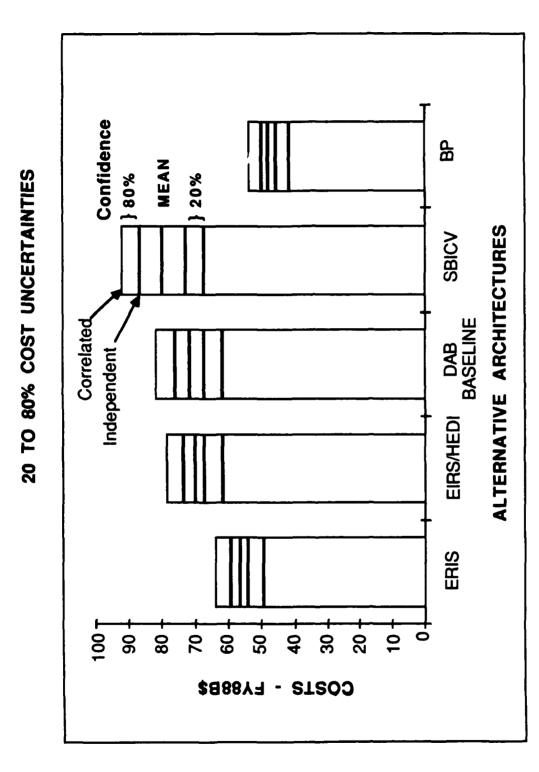


Figure 7. Comparison of Program Cost Ranges

#### III APPROACH USING LOG-NORMAL DISTRIBUTIONS

The assumptions implicit in our approach in this section of the report are the following:

The costs (random variables) for each phase (R&D and Investment) for each element in an architecture are independent.

The probability function for the cost of a phase for an element has a lognormal distribution.

The nature of the cost distribution is constant by phase, that is, the cost risk distribution (probability of a cost) for an element in R&D is similar to the risk distribution for deployment (investment). We will represent this assumption by saying that the coefficient of variation remains constant.

The elements are divided into three categories: those with high risk, those with moderate risk, and those with low risk. Within a constant risk category, the coefficient of variation is the same.

Important relationships for the log-normal density function are given in Table 6.

Table 6.

Log-Normal Distribution Definitions

 $f_X(x) =$ 

**Density Function** 

$$f_X(x) = \exp \left[-1/2 (\ln x - \mu)^2 / \sigma^2\right] / \left[\sqrt{(2\pi)} \sigma x\right]$$
  $o < x < \infty$   $\mu > o$ 

 $\sigma > 0$ 

Cumulative Distribution Function

$$F_X(x) = \Phi[(\ln x - \mu) / \sigma]$$

Fifty Percent Confidence Value

$$X_{50}=\exp(\mu)$$

Expected Value

$$E(X) = \exp[\mu + (1/2)\sigma^2]$$

Variance

$$VAR(X) = { exp[2\sigma^2] - exp[\sigma^2] } exp[2\mu]$$

Mode

$$x^* = \exp{(\mu - \sigma^2)}$$

Coefficient of Variation

$$\sqrt{VAR(X)/E(X)} = \sqrt{[\exp(\sigma^2) - 1]}$$

In order to fit the log-normal distribution to the input data, we must find values for  $\mu$  and  $\sigma$ . On each figure of Figures 1-3, we are given the 50% confidence value. Its natural logarithm is equal to  $\mu$ . To find the value of  $\sigma$ , we derived the normal equation for its regression only to find, after simplifications, the intractable result

$$\delta D/\delta \sigma = 0$$

$$n$$

$$= \sum_{i=1}^{\infty} [\Phi\{\Theta_i\} - P_i] \exp(-(1/2)\Theta_i^2) \Theta_i$$

where D is the sum of the squares of the differences between the log-normal distribution to be fit and the probabilities that were taken from the curves,  $\Theta_i$  is equal to  $[\log(x_i - \mu]/\sigma, x_i]$  is the ith cost point and  $P_i$  is the corresponding probability from the figure. To find the value of  $\sigma^*$ , the optimal estimate to minimize the differences, we searched over the values of  $\sigma$ . The results are given in Table 7. The data extracted from Figures 1-3 and used in the calculations leading to the values of  $\sigma$  are contained in Appendix B.

Table 7
Log-Normal Fits to Input Data

System	μ	σ	√VAR/E (x)
BSTS	ln(7.9)	0.12308	0.12355
SSTS	ln(8.7)	0.18352	0.18508
SBI	ln(14.3324)	0.28108	0.28672

Figure 8 compares the log-normal curve fit to the input data extracted from Figure 1 for the BSTS. Figures 9 and 10 similarly compare the curve fits to the data of Figures 2 and 3 for the SSTS and the SBI respectively.

We have characterized these three systems as representing three classes of risk: low, moderate and medium. We have applied these risk categories to each of the elements in the set of architectures investigated in ACES. The results of that application are given in Table 8.

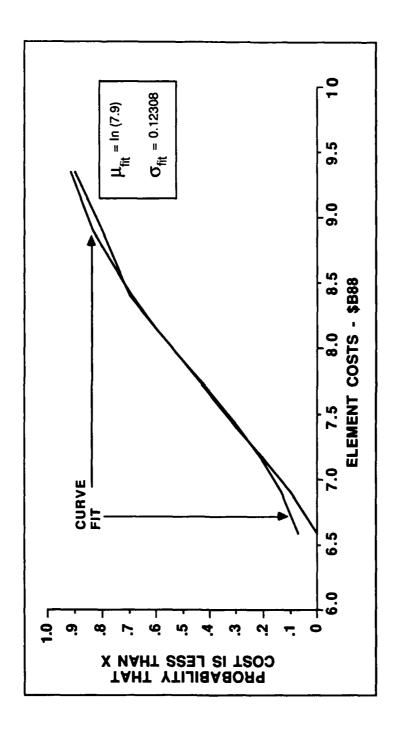


Figure 8. Comparison of Fitted Log Normal Distribution with Input Curve for BSTS

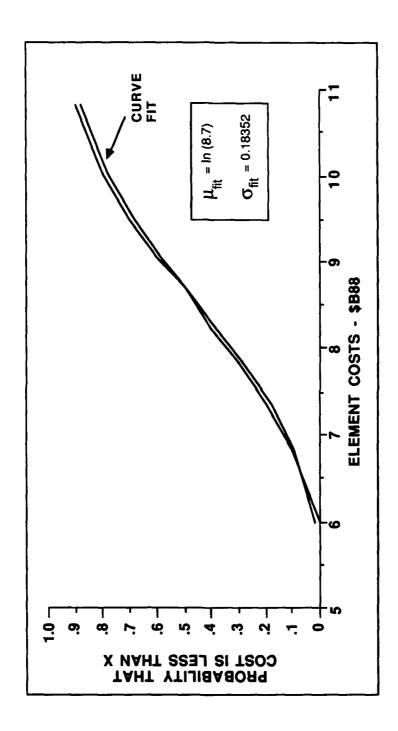


Figure 9. Comparison of Fitted Log Normal Distribution with Input Curve for SSTS

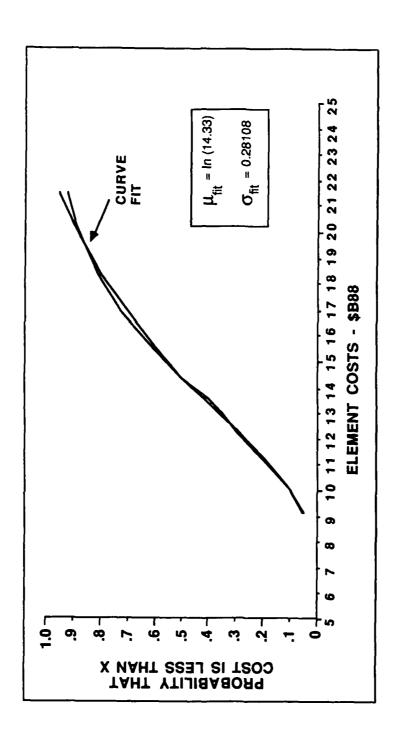


Figure 10. Comparison of Fitted Log Normal Distribution with Input Curve for SBI

Table 8
Risk Categories Assumed

System	System	
Low Risk Category	Moderate Risk Category	
BSTS	SSTS	
GBR	GSTS	
ERIS		
SEI	High Risk Category	
LAUNCH	SBI	
	CC/SOIF	

Using these data and assumptions, we can now develop the total costs for each of the architectures. For the log-normal distributions, we have two procedures. In the first, we will take the input costs we have for R&D and Investment to be the 50% confidence estimates for each phase for each of the elements, devise their distributions according to the risk categories, and then sum those costs according to the rules we have developed for estimating the convolution of log-normal distributions. In the second procedure, we will take the input costs we have for R&D and Investment to be the expected value estimates for each phase for each element, and then proceed in the same manner as above.

In the first procedure, for each phase for the ith element, we assume that the inputs are C<sub>50</sub>, the 50% confidence cost for the respective R&D and investment costs, and cv, the coefficient of variation for that element. We derive the parameters for the corresponding log-normal distribution as

$$\mu_i = \ln(C_{50})$$

$$\sigma_i = \sqrt{\ln(1+cv^2)}$$

For example, if the nominal costs for BSTS R&D are 5.4B\$, if that 5.4B\$ is equivalenced to  $C_{50}$ , and if it is in a low risk category, then

$\mu_1 =$	$\ln(5.4) = 1.6864$	Derived from Nominal Costs
cv =	0.12308	Taken from Risk Category
σ <sub>1</sub> =	$\sqrt{[\ln(1+cv^2)]}$	Derived from cv

In the second procedure, for each phase for the ith element, we assume that the inputs are E(X), the expected value for the respective R&D and

investment costs, and cv, the coefficient of variation for that element. We derive the parameters for the corresponding log-normal distribution as

$$\sigma_i = \sqrt{\ln(1+cv^2)}$$
  
$$\mu_i = \ln[E(X)] - \sigma_i^2/2$$

For example, if the nominal costs for BSTS R&D are 5.4B\$, if that 5.4B\$ is equivalenced to E(X), and if it is in a low risk category, then

cv	=	0.12308	Taken from Risk Category
$\sigma_1$	=	√[ln(1+cv²)] 0.1226	Derived from cv
$\mu_1$		ln(5.4) - σ <sub>i</sub> <sup>2</sup> /2 1.6789	Derived from Nominal Costs

For the log-normal distribution, we approximate the convolutions by assuming that the convolution will yield (see Appendix A for justificiation of this approximation) a log-normal like distribution characterized by parameters whose values are derived from the sum of the expected values and variances. The algorithm is given as follows:

$$E(X) = \sum E(X_i)$$

$$\alpha = \ln[E(X)]$$

$$\beta = VAR(X) = \sum VAR(X_i)$$

$$\mu = 1/2\{4\alpha - \ln[\beta + \exp(2\alpha)]\}$$

$$\sigma = \sqrt{[2(\alpha - \mu)]}$$

where a and  $\beta$  are intermediate values, and  $\mu$  and  $\sigma$  are the parameters of the resultant log-normal distribution for the total system costs.

We will use this algorithm to combine the costs for the R&D phase and the Investment phase, and to combine the costs for each of the elements. We will first develop the acquisition costs as the sum of the R&D and Investment costs for the elements that are deployed in an architecture. Then, we will develop the program costs as the sum of the R&D costs for all of the elements being considered for Phase One, either for immediate or slightly later deployment, and the investment costs for the elements to be immediately deployed.

#### **RESULTS**

Assuming the first procedure, that is, nominal costs are 50% confidence estimates, Table 9 presents a summary of the costs for each of the architectures. The supporting data tables presenting the sums of expected values and variances are illustrated in Appendix C for the DAB Architecture.

Table 9
Summary of Acquisition and Program Costs Derived with Independent Log-Normal Distributions
Assuming Inputs are 50% Confidence Values of Cost

	210044114	ing imputes the our	Commen	ce varaes or c	-001	
		Acquis	ition Costs			
Probability	у					
		Archi	itectures	_		
	<b>ERIS</b>	ERIS/HEDI	DAB	SBICV	BP	
0.20	45	62	66	65	37	
0.50	47	64	70	<i>7</i> 2	39	
0.80	49	66	74	79	40	
		Progr	am Costs			
Probability	y					
		Arch	itectures			
	<b>ERIS</b>	ERIS/HEDI	DAB	SBICV	BP	
0.20	55	69	69	<b>7</b> 5	46	
0.50	58	<b>7</b> 1	<i>7</i> 3	81	48	
0.80	60	<i>7</i> 4	<i>7</i> 7	88	50	

Assuming the second procedure, that is, nominal costs are expected values, Table 10 presents the results of our calculations. The supporting data tables presenting the sums of expected values and variances are illustrated in Appendix D for the DAB Architecture.

Table 10
Summary of Acquisition and Program Costs Derived with Independent Log-Normal Distributions
Assuming Inputs are Expected Values of Cost

		Acquis	ition Costs			_
Probabilit	у					
		Arc	hitectures			
	<b>ERIS</b>	ERIS/HEDI	DAB	SBICV	BP	
0.20	44	61	64	64	36	
0.50	46	63	68	70	38	
0.80	48	65	72	77	40	
Program (	Costs					
Probabilit	у					
		Ar	chitectures			
	<b>ERIS</b>	ERIS/HEDI	DAB	SBICV	BP	
0.20	54	68	67	<i>7</i> 3	46	
0.50	57	<b>7</b> 0	<b>7</b> 1	<i>7</i> 9	47	
0.80	<del>59</del>	73	<i>7</i> 5	86	49	

#### IV APPROACH USING NORMAL DISTRIBUTIONS

In this section, we derive the system costs assuming normal distributions as the distributions underlying the risk assessment data. The assumptions implicit in our approach are the following:

The costs (random variables) for each phase (R&D and Investment) for each element in an architecture are independent.

The probability function for the cost of a phase for an element has a normal distribution.

The nature of the cost distribution is constant by phase. We will represent this assumption by saying that the coefficient of variation remains constant.

The elements are divided into three categories: those with high risk, those with moderate risk, and those with low risk. Within a constant risk category, the coefficient of variation is the same.

Important relationships for the normal density function are given in Table 11.

Table 11
Normal Distribution Definitions

Normal Distribution Definitions		
Density Function		
$f_X(x) = \exp[-(1/2)(x - \mu)^2/\sigma^2] / [\sqrt{(2\pi)} \sigma]$	-∞ < x < ∞	
•	-∞ < µ < ∞	
	σ>0	
Cumulative Distribution Function		
$F_{x}(x) = \Phi[(x-\mu) / \sigma]$		
Fifty Percent Confidence Value		
$X_{50} = \mu$		
Expected Value		
$E(X) = \mu$		
Variance		
$VAR(X) = \sigma^2$		
Mode		
$x^* = \mu$		
Coefficient of Variation		
$\sqrt{VAR(X)/E(X)} = \sigma/\mu$		

In order to fit the normal distribution to the input data, we must find values of  $\mu$  and  $\sigma$ . On each figure of Figures 1-3, we are given the 50% confidence value. It is equal to  $\mu$ . To find the value of  $\sigma$ , we derived the normal equation for its regression only to find, after simplifications, the intractable result

$$\delta D/\delta \sigma = 0$$

$$n$$

$$= \sum_{i=1}^{n} {\{\Phi\{\Theta_i\} - P_i\} \exp(-(1/2)\Theta_i^2)}$$

where D is the sum of the squares of the differences between the normal distribution to be fit and the probabilities that were taken from the curves,  $\Theta_i$  is equal to  $[x_i - \mu]/\sigma$ ,  $x_i$  is the ith cost point and  $P_i$  is the corresponding probability from the figure. To find the value of  $\sigma^*$ , the optimal estimate to minimize the differences, we searched over the values of  $\sigma$ . The results are given in Table 12. The data extracted from Figures 1-3 and used in the calculations leading to the values of  $\sigma$  are contained in Appendix B.

Table 12 Normal Fits to Input Data

System	μ	σ	σ/μ
BSTS	7.9	0.97241	0.1231
SSTS	8.7	1.58450	0.1821
SBI	14.3324	3.99300	0.2786

Figure 11 compares the normal curve fit to the input data extracted from Figure 1 for the BSTS. Figures 12 and 13 similarly compare the curve fits to the data of Figures 2 and 3 for the SSTS and the SBI respectively.

Using these data and assumptions, we can now develop the total costs for each of the architectures. For each phase for the ith element, we assume that the inputs are  $C_{50}$ , the 50% confidence cost for the respective R&D and investment costs, and cv, the coefficient of variation for that element. We derive the parameters for the corresponding normal distribution as

$$\mu_i = C_{50}$$

$$\sigma_i = \mu_i \text{ cv}$$

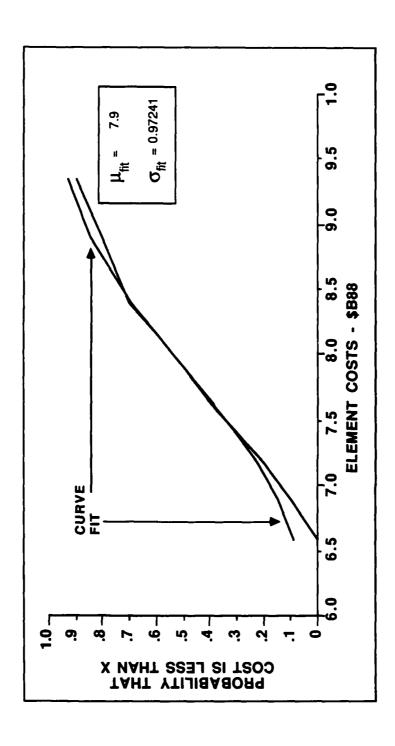


Figure 11. Comparison of Fitted Normal Distribution with Input Curve for BSTS

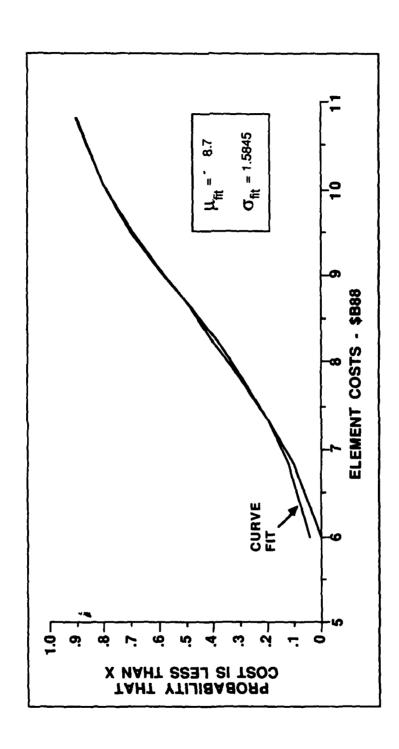


Figure 12. Comparison of Fitted Normal Distribution with Input Curve for SSTS

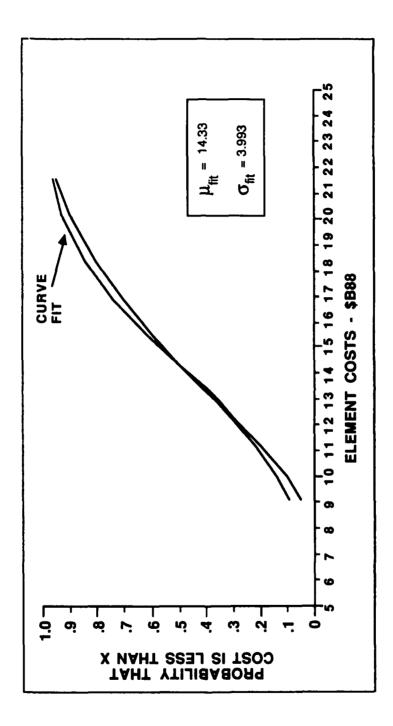


Figure 13. Comparison of Fitted Normal Distribution with Input Curve for SBI

For example, if the nominal costs for BSTS R&D are 5.4B\$, if that 5.4B\$ is equivalenced to  $C_{50}$ , and if it is in a low risk category, then

 $\mu_1 = 5.4$ Derived from Nominal Costs cv = 0.1231Taken from Risk Category  $\sigma_1 = \mu_1 cv$ Derived from cv = 0.6647

For the normal distribution, we have a closed form expression for the convolutions, which yield a normal distribution with the values for the two parameters being

$$\mu = E(X) = \sum E(X_i)$$

$$\sigma^2 = VAR(X) = \sum VAR(X_i)$$

We will use this algorithm to combine the costs for the R&D phase and the Investment phase, and to combine the costs for each of the elements.

Table 13 presents the results of our calculations. The supporting data tables illustrating the sums of expected values and variances are presented in Appendix E for the DAB Architecture.

Table 13
Summary of Acquisition and Program Costs Derived with Independent Normal Distributions

		macpenaem	110111111111111111111111111111111111111	Still ations		
		Acquis	ition Costs			
Probability	/					
		Arc	hitectures			
	ERIS	ERIS/HEDI	DAB	SBICV	BP	
0.20	44	61	64	64	36	
0.50	46	63	68	70	38	
0.80_	48	65	72	77	40	
		Progr	am Costs			
Probability	7				-	
		Arc	hitectures			
	<b>ERIS</b>	ERIS/HEDI	DAB	SBICV	BP	
0.20	54	68	68	<b>73</b>	46	
0.50	<i>57</i>	<i>7</i> 0	<i>7</i> 2	80	48	
0.80	59	<b>73</b>	<i>7</i> 5	86	50	

#### V SENSITIVITIES TO ASSUMPTIONS

Recognizing that the range of values about the 50% confidence values is a lot less than we are accustomed to seeing, we began to examine some of our assumptions.

#### DIFFERENTIATING RISK ACCORDING TO PROGRAM PHASE

The first assumption we will investigate is the assumption that the risk distribution for R&D and Investment is the same. We will examine here the consequences of making an alternative assumption.

In developing a cost breakdown, the USA generally assigns some 15% of R&D and some 5% of the investment costs to risk mitigation. We will use these estimates to derive the variances of R&D and Investment programs from the coefficients of variations obtained from the USAF input data on total programs. (See Figures 1 through 3.) The problem is to find the standard deviations for an R&D program,  $\sigma_1$ , and Investment program,  $\sigma_2$ , when we know the expected values,  $\mu_1$  and  $\mu_2$  of these two phases, and when we know the coefficient of variation of the entire program. Seeing as how our disparate assumptions about the underlying distributions hardly affect our answers, we will work with the normal distribution since that is the easiest one to use. We can now state the problem as

Find  $\sigma_1$  and  $\sigma_2$  given  $\mu_1$ ,  $\mu_2$ , cv, and the relationships

$$cv = \sigma/\mu$$

$$\sqrt{(\sigma_1^2 + \sigma_2^2)}$$

$$\mu_1 + \mu_2$$

and

$$\sigma_1 / \mu_1 = 3 \, \sigma_2 / \mu_2$$

where the unsubscripted variables apply to the combined program data.

A little algebra yields

$$\sigma_1 = 3 \sigma_2 \mu_1 / \mu_2$$

and

$$\sigma_2^2 = \frac{[cv (\mu_1 + \mu_2)\mu_2]^2}{9 \mu_1^2 + \mu_2^2}$$

Replacing these different estimates of variation for the estimates that assumed both phases of a program were equal yields no significant difference. In Appendix F, we present the results for the DAB Architecture assuming the values of  $\sigma 1$  and  $\sigma 2$  defined above. Table 14 compares those data with the data obtained from Table 13.

Table 14
Comparison of Architecture Cost Ranges for
Two Different Assumptions About the Split Between
Program Phase Cost Variations

Acquisition Costs Prob	Assuming Equal Risks	Assuming 3:1 Risk Ratio	Differences	
0.20	64.46	63.36	1.10	
0.50	68.31	68.31	0.00	
0.80	72.16	73.26	-1.10	
Program Costs Prob	Assuming Equal Risks	Assuming 3:1 Risk Ratio	Differences	
0.20	67.71	66.58	1.13	
0.50	<i>7</i> 1.59	71.59	0.00	
0.00				

# COMPUTING COST ESTIMATES ASSUMING PROGRAM INTER-DEPENDENCIES

The second assumption we will investigate is independence. In the earlier sections, we assumed that the costs for the phases of a program element, that is, the cost distributions for the R&D and Investment phases, were independent one from another, and that all of the program elements were also independent one from another. It is to these assumptions that we owe the tractability of analysis of the log-normal distributions and the ease of the analysis for the normal distribution. Here, in this subsection, we examine possible implications of interdependencies by assuming that the program phases and the program elements may not be independent.

IMPACT OF CORRELATION WITHIN A PROGRAM. Assuming that the underlying distributions are normal, but correlated, we derive the distribution for the sum of the R&D cost and the Investment cost and factor that into the total system costs. To do so, we further assume that the correlation coefficient,  $\rho$ , is known. If we let  $X_1$  be the random R&D costs,

normally distributed with mean  $\mu_1$  and variance  $\sigma_1$  and  $X_2$  be the random Investment costs, normally distributed with mean  $\mu_2$  and variance  $\sigma_2$ , and  $Z_i$ , their sum for the ith program element, then  $Z_i$  is normally distributed with parameters

$$E(Z_i) = \mu_1 + \mu_2$$
VAR  $(Z_i) = \sigma_1^2 + \sigma_2^2 + 2\rho\sigma_1\sigma_2$ 

Provided that we make the assumption that we know the mean and the variance of the distributions for the costs of the two phases, (actually determined for this exercise by analogy to Figures 1-3 and through the coefficient of variation) then we can first compute the distributions of the costs for the individual programs, taking into account the correlations, and then compute the total system cost distributions (assuming still that the program elements are independent). For the DAB Architecture, the 20% confident, 50% confident and 80% confident costs are presented in Figure 14 as a function of the assumed correlation coefficient. (In generating this plot, note that we have assumed that the correlation coefficient between R&D and Investment is the same for all program elements.)

IMPACT OF PROGRAM CORRELATION. Assuming that the underlying program element distributions are normal, but that they are correlated with one another, we can also derive the distribution for the total system costs. To do this we must know the covariance matrix, that is, the correlations of the cost distributions for each program element with the cost distributions of each of the other program elements in the system. As above, we will assume that we know the covariance matrix. For our sensitivity analysis, we will assume that the correlation coefficient between any two programs is identical to  $\rho$ , that is,  $COV[Zi,Zj] = \rho\sigma i\sigma j$ . If we let Z be the total system costs, then Z is normally distributed with parameters

$$E(Z) = \sum_{i=0}^{n} E(Z_i)$$

$$i=0$$

$$n$$

$$VAR(Z) = \sum_{i=0}^{n} \sum_{j=0}^{n} COV[Z_i, Z_j]$$

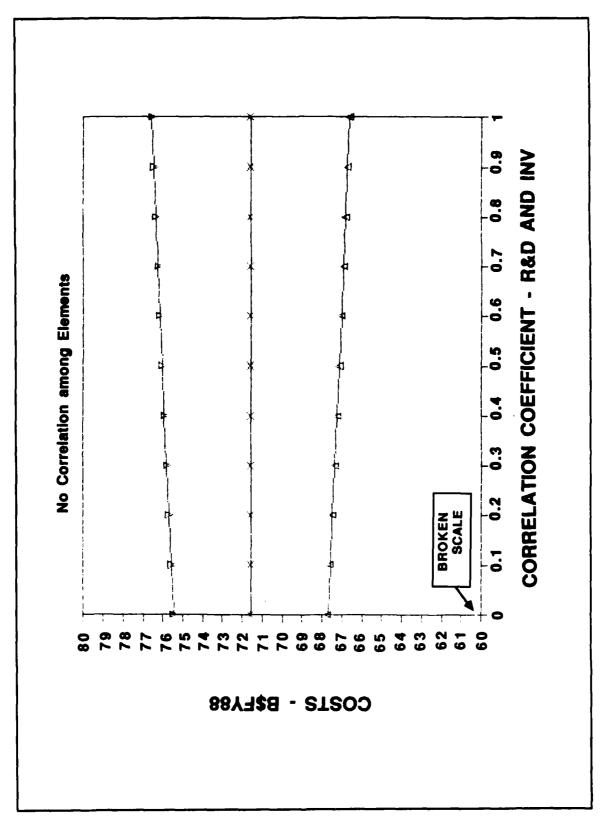


Figure 14. Program Costs for DAB Architecture

where n is the number of program elements whose costs are to be summed for the acquisition or program costs.

For the DAB Architecture, we plot in Figure 15 the 20%, 50% and 80% confident costs for the total acquisition costs and in Figure 16 the same quantities for the total program costs.

For the calculations in Figures 15 and 16, we have assumed that the correlation coefficient between R&D and Investment cost distributions is equal to 0.50.

Just as a further point of reference, consider the following comparison. For a correlation coefficient of unity for the correlation between programs and a correlation coefficient of 0.5 for the correlation between R&D and Investment, the program costs were estimated at 81B\$. If we assumed unity for both correlation coefficients, the program costs were estimated at 82.1B\$.

Just as a bound to use with these results, consider the cost of the acquisition for the DAB architecture, derived assuming that the total acquisition costs had the same coefficient of variation we assumed for the SBI program. That cv was 0.2786. Given the mean value of the acquisition as 68B\$, the standard deviation should be 18.95 B\$. AT 20% confidence, the acquisition costs we calculate should be more than 52.1B\$. AT 80% confidence, the acquisition costs should be less than 84.0B\$. Obviously, the 50% confident number is still 68B\$. Note that the costs we have derived in Figure 15 for the acquisition costs for the fully correlated case are within those bounds.

COST ESTIMATES WITH LARGER VARIANCES. We note that the costs bounds just developed, assuming full correlations between programs, are still tight relative to what might be expected. For a program that is at this stage of its development and still requires advances in technology, some might expect that the cost ranges should be broader than that, for example, the upper value of the range being something like twice the lower value. However, the procedure we have applied here seems consistent with what we know, and uses our current best estimates of the variations.

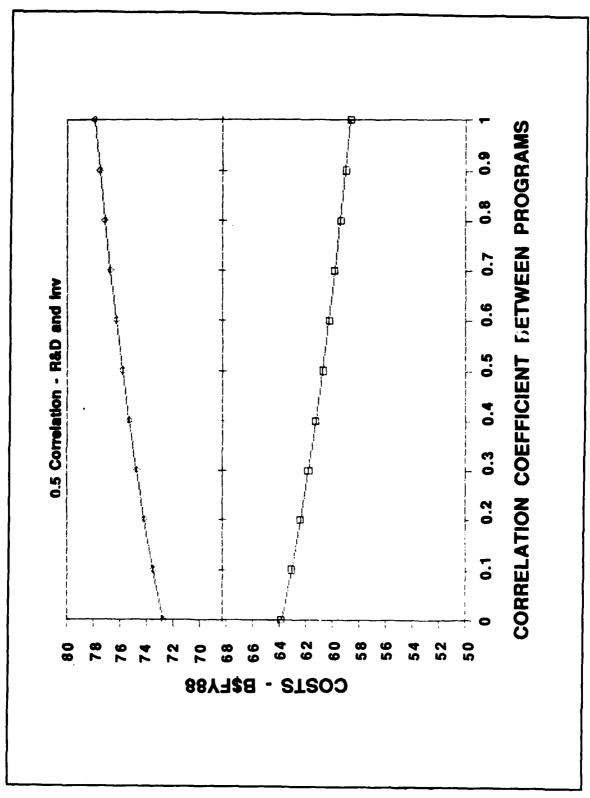


Figure 15. Acquisition Costs for DAB Architecture

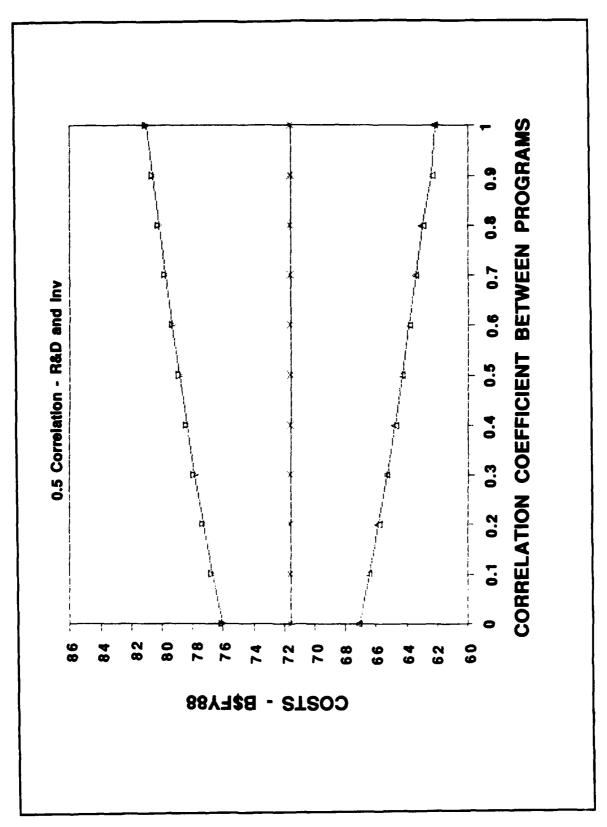


Figure 16. Program Costs for DAB Architecture

We have experimented (made mistakes in our inputs) with larger variances for the log-normal distributions and obtained instances where the range of values (ratio of 80% confident values to 20% confident values) did have close to a factor of two between them (for the DAB architecture, we manufactured a case where the lower -- 20% confidence -- was 62B\$ and the upper bound -- 80% confidence -- was 109B\$). However, the variances we used to get those results were much larger than any we are currently deriving from Figures 1-3. Just to illustrate, the coefficient of variation we used to obtain the almost 2:1 range with the log-normal distribution were, for the low risk elements, 0.1215, for the moderate risk elements, 0.3031, and for the high risk elements, 1.2258. These can be compared with the coefficient of variation we are now using with the log-normal distribution: 0.12136 for the low risk elements, 0.1851 for the moderate risk elements, and 0.2867 for the high risk elements. There had to be a factor of two increase in the variation for the moderate risk elements and a factor of four increase in the variation of the high risk elements for us to find a value close to the 2:1 ratio between the upper and lower bounds.

If the cost ranges are to reflect a higher ratio than we have found here in this report, then the input estimates of variation must increase. This, of course, will not set well with program managers. On the other hand, the cost variations we include do not include the many sources of variation that come from outside a program manager's control, but still typically increase the costs of a program element: politically inspired delays, mission changes and threat modifications. Incorporating these variations, if we could, would significantly increase the variation, and the range of estimates we would find.

# Appendix A

**Log-Normal Distributions** 

# Appendix A

# Log-Normal Distributions

The purpose of this appendix is to present some features of the log-normal distribution. The notation is taken from Ref A-1.

Though there are a number of alternative expressions for the density function of the log-normal distribution, the form of the <u>log-normal density function</u> we will use here is given by

$$f_X(x) = \exp[-(1/2)(\ln x - \mu)^2/\sigma^2] / [\sqrt{(2\pi)} \sigma x]$$
 o < x < \infty

This density function is also a function of the values of the two parameters,  $\mu$  and  $\sigma$ , whose features we will explore below.

The log-normal cumulative distribution function is then

$$F_X(x) = \int_0^x \exp[-(1/2) (\ln y - \mu)^2 / \sigma^2] \, dy / [\sqrt{(2\pi)} \, \sigma y]$$

We can transform this equation by substituting

$$u = (\ln y - \mu)/\sigma$$
$$du = dy/(\sigma y)$$

to obtain

$$F_X(x) = \int_{-\infty}^{(\ln x - \mu)/\sigma} \exp[-(1/2)u^2] du / \sqrt{(2\pi)}$$

which is equivalent to

$$F_X(x) = \Phi [(\ln x - \mu) / \sigma]$$

where  $\Phi(x)$  is the cumulative standard normal function.

The Expected Value of x, E(X), is given by

$$E(X) = \int_{0}^{\infty} x \exp[-(1/2) (\ln x - \mu)^{2} / \sigma^{2}] dx / [\sqrt{(2\pi)} \sigma x]$$

### Again substituting

$$u = (\ln x - \mu)/\sigma$$

$$du = dx/(\sigma x)$$

$$exp(\sigma u + \mu) = x$$

we obtain

$$E(X) = \int_{-\infty}^{\infty} \exp(\sigma u + \mu) \exp[-(1/2)u^{2}] du / \sqrt{(2\pi)}$$

$$E(X) = \int_{-\infty}^{\infty} \exp[-(1/2)(u^{2} - 2\sigma u - 2\mu)] du / \sqrt{(2\pi)}$$

$$E(X) = \int_{-\infty}^{\infty} \exp[-(1/2)(u^{2} - 2\sigma u + \sigma^{2} - \sigma^{2} - 2\mu)] du / \sqrt{(2\pi)}$$

$$E(X) = \int_{-\infty}^{\infty} \exp[-(1/2) \{(u - \sigma)^2 - (\sigma^2 + 2\mu)\}] du / \sqrt{(2\pi)}$$

$$E(X) = \int_{-\infty}^{\infty} \exp[-(1/2)(u - \sigma)^2] \exp[(1/2)(\sigma^2 + 2\mu)] du / \sqrt{(2\pi)}$$

$$E(X) = \exp[(1/2)(\sigma^2 + 2\mu)] \int_{-\infty}^{\infty} \exp[-(1/2)(u - \sigma)^2] du / \sqrt{(2\pi)}$$

$$E(X) = \exp[(1/2)(\sigma^2 + 2\mu)]$$

The Second Moment of x,  $E(X^2)$ , is given by

$$E(X^{2}) = \int_{0}^{\infty} x^{2} \exp \left[-(1/2) (\ln x - \mu)^{2} / \sigma^{2}\right] dx / \left[\sqrt{(2\pi)} \sigma x\right]$$

### Again substituting

$$u = (\ln x - \mu)/\sigma$$
$$du = dx/(\sigma x)$$
$$exp(\sigma u + \mu) = x$$

we obtain

$$\begin{split} E(X^2) &= \int_{-\infty}^{\infty} & \exp[2(\sigma u + \mu)] \exp[-(1/2)u^2] \, du \, / \, \sqrt{(2\pi)} \\ E(X^2) &= \int_{-\infty}^{\infty} & \exp[-(1/2)\{u^2 - 4\sigma u - 4\mu\}] \, du \, / \, \sqrt{(2\pi)} \\ E(X^2) &= \int_{-\infty}^{\infty} & \exp[-(1/2)\{u^2 - 4\sigma u + 4\sigma^2 - 4\sigma^2 - 4\mu\}] \, du \, / \, \sqrt{(2\pi)} \\ E(X^2) &= \int_{-\infty}^{\infty} & \exp[-(1/2)(u - 2\sigma)^2] \exp[2\sigma^2 + 2\mu] \, du \, / \, \sqrt{(2\pi)} \\ E(X^2) &= \exp[2\sigma^2 + 2\mu] \, \int_{-\infty}^{\infty} & \exp[-(1/2)(u - 2\sigma)^2] \, du \, / \, \sqrt{(2\pi)} \\ E(X^2) &= \exp[2\sigma^2 + 2\mu] \end{split}$$

The Variance of X is given by

VAR(X) = E(X<sup>2</sup>) - E<sup>2</sup>(X)  
= 
$$\exp[2\sigma^2 + 2\mu] - \exp[\sigma^2 + 2\mu]$$
  
=  $\{\exp[2\sigma^2] - \exp[\sigma^2]\} \exp[2\mu]$ 

The  $\underline{Mode}$  of the distribution is obtained by finding the value of x which makes the derivative of the density function equal to zero. Setting

$$f'_X(x) = -\sqrt{(2\pi)} \{ (\ln x - \mu)/\sigma + \sigma \} \exp[-(1/2)(\ln x - \mu)^2/\sigma^2] = 0$$

we have for the mode at x

$$(\ln x^* - \mu)/\sigma = -\sigma$$

or

$$x^* = \exp{(\mu - \sigma^2)}$$

The <u>coefficient of variation</u> is defined as the ratio of the standard deviation ( $\sqrt{VAR(X)}$ ) to the expected value (E(X)). We are using it here as a standard deviation normalized by the mean. We illustrate in Figure A-1 the similarity of five different log-normal distributions which have the the same coefficient of variation, but have with values of  $\exp(\mu)$  ranging from 5 to 9 in steps of one. The similarity between the resulting cumulative distributions is obvious. Though the  $\sigma$  parameter remains constant, the variance does not: it increases as  $\mu$  increases.

Using the multiplicative property of exponents, the square root of the variance becomes

$$\sqrt{VAR(X)} = \exp(\mu)[\exp(2\sigma^2) - \exp(\sigma^2)]^{(1/2)}$$

Expressing the expected value as a product of two exponential terms, we can rewrite it as

$$E(X) = \exp(\mu) \exp(\sigma^2/2) = \exp(\mu) \left[\exp(\sigma^2)\right]^{(1/2)}$$

The ratio of the two equations above yields for the coefficient of variation, cv,

$$\sqrt{VAR(X)/E(X)} = \{ [\exp(2\sigma^2) - \exp(\sigma^2) ] \exp(-\sigma^2) \}^{(1/2)}$$
$$= [\exp(\sigma^2) - 1]^{(1/2)} = cv$$

This equation can be turned around to provide a value for  $\sigma$  if the coefficient of variation is known.

$$\sigma = \sqrt{\ln(1+cv^2)}$$

This equation does not have an explicit dependence on  $\mu$ .

We note in Table A-1 the value of the log normal cumulative distribution function for several of the key values of x.

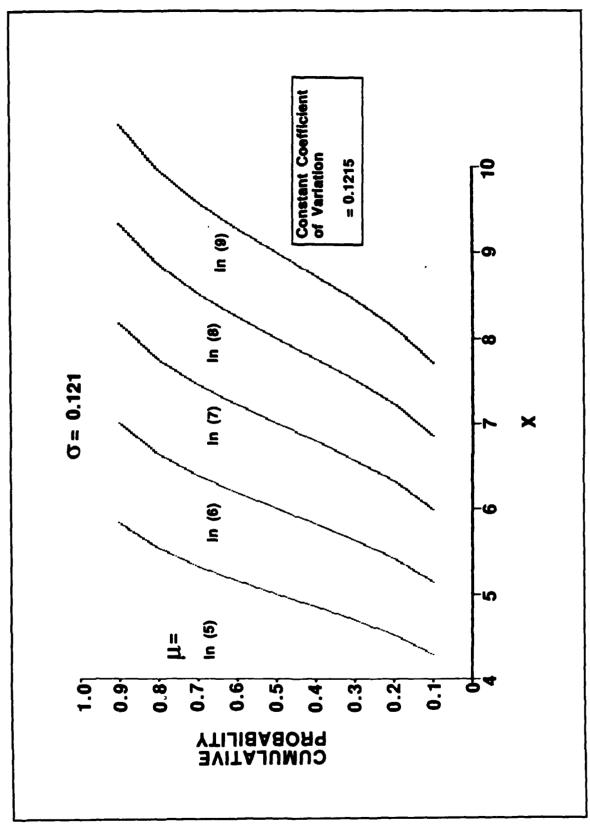


Figure A-1. Comparison of Log Normal Risk Distributions

Table A-1
Log-Normal Function Values

x	F <sub>X</sub> (x)	
$E(X)=\exp[\mu+(1/2)\sigma^2]$	Φ(-σ/2)	
Mode(X)= $\exp(\mu - \sigma^2)$	Φ(-σ)	
exp(μ-1.2801σ)	0.10	
exp(μ-0.8391σ)	0.20	
exp(μ-0.5216σ)	0.30	
$\exp(\mu - 0.2513\sigma)$	0.40	
exp(μ)	0.50	
exp(μ+0.8391σ)	0.80	
exp(μ+1.6445σ)	0.95	
exp(μ+2.3276σ)	0.99	

If I have two independent random variables with log-normal distributions, say  $X_1$  and  $X_2$ , and I want to determine the <u>distribution of the sum of two log-normals</u>,  $Z = X_1 + X_2$ , I could proceed with either the method of convolutions or of characteristic functions. However, since I could make neither of these methods yield tractable expressions, I have fallen back on the following approximation: I have assumed that the resulting distribution has a log-normal distribution with parameters  $\mu$  and  $\sigma$  determined from the distributions of the variables to be added together. I now need only to determine those values.

Since I have assumed that the distributions are independent, I know the following to be true

$$E(Z) = E(X_1) + E(X_2)$$

$$VAR(Z) = VAR(X_1) + VAR(X_2)$$

Since I have assumed that all distributions are log-normal, I also know the dependencies of each of these quantities on their respective parameters. Thus, for the sum of two log-normal distributions, I can write

$$\mathrm{E}(Z) = \exp(\mu + (1/2)\sigma^2) = \exp(\mu_1 + (1/2)\sigma_1^2) + \exp(\mu_2 + (1/2)\sigma_2^2) = \mathrm{A}$$

$$\begin{split} VAR(Z) &= \exp(2\mu)[\exp(2\sigma^2) \text{-} \exp(\sigma^2)] \\ &= \exp(2\mu_1)[\exp(2\sigma_1^2) \text{-} \exp(\sigma_1^2)] \\ &\quad + \exp(2\mu_2)[\exp(2\sigma_2^2) \text{-} \exp(\sigma_2^2)] \\ &= \beta \end{split}$$

The subscripted variables are assumed known and fixed and thus the expressions yield constants, A and  $\beta$ , from which we must determine  $\mu$  and  $\sigma$ . We can write  $\sigma$  in terms of  $\mu$  by taking the logarithm of both sides of the first equation and manipulating the result:

$$\ln[\exp(\mu + (1/2)\sigma^2)] = \ln[A] = \alpha$$
$$\mu + (1/2)\sigma^2 = \alpha$$
$$\sigma^2 = 2(\alpha - \mu)$$

Substituting for  $\sigma^2$  in the expression for the variance of Z we obtain

$$\exp(2\mu) \left[ \exp\{4(\alpha-\mu)\} - \exp\{2(\alpha-\mu)\} \right] = \beta$$

Multiplying through by the exponent outside the brackets on the left

$$\exp\{4\alpha-2\mu\} - \exp\{2\alpha\} = \beta$$

Taking the logarithm of both sides

$$\{4\alpha-2\mu\}=\ln(\beta+\exp\{2\alpha\})$$

Manipulating the resulting expression

$$\mu = (1/2)[4\alpha - \ln(\beta + \exp\{2\alpha\})]$$

With  $\mu$  known, we can also solve for  $\sigma$ .

We illustrate the quality of the approximation with several <u>examples of distributions of sums of log-normals</u>. Let us assume the two log-normal variables have distributions with the parameters given in Table A-2.

Table A-2
Parameter Values for First Example

Parameter	$X_1$	X <sub>2</sub>
μi	ln8	ln5
$\sigma_{\mathbf{i}}$	1	1
E(X)	13.19	8.24
VAR(X)	298.93	116.77
Α		21.43
α		3.06
β		415.70
μ		$2.743 = \ln 15.52$
σ		0.803

To test the quality of the approximation, we show in Figure A-2 the convolution of the two distributions (computed numerically) with the curve generated by the approximation. As it turns out, the approximation looks fair. This quality of fit suggests that there is something very much like a lognormal result to the convolution.

We tried a second example as well. The parameters are given in Table A-3. The two products, one a numerical convolution and the other the product of our approximation, are presented in Figure A-3. Again, the fit is very close.

Table A-3
Parameter Values for Second Example

Parameter	X <sub>1</sub>		X <sub>2</sub>
$\mu_{i}$	ln8		ln5
$\sigma_{\mathbf{i}}$	0.50		0.50
E(X)	9.06		5.67
VAR(X)	23.34		9.12
Α		14.73	
αα		2.69	
β		32.46	
μ		$2.620 = \ln 13.739$	
σ		0.373	

Finally, in Table A-4, we present an example that approximates some of the values assumed for  $\sigma_i$  in the body of the report. In Figure A-4, we compare the approximation with the numerical convolution. The result is a perfect fit within the resolution of the graph.

Table A-4
Parameter Values for Third Example

Parameter	X <sub>1</sub>	X <sub>2</sub>
μi	ln8	ln5
$\sigma_{ m i}$	0.10	0.10
E(X)	8.04	5.03
VAR(X)	0.65	0.25
Α		13.06
a		2.57
β		0.90
μ		$2.567 = \ln 13.03$
σσ		0.077

The three examples given above suggest (and do no more than that) that our approximation is good enough for an estimate of the convolution of two log-normal distributions. When we are adding the results from several, we will assume that the approximation holds good. However, we could also assume that the sum of a large number of log-normal distributions will result in an approximately normal distribution by the central limit theorem.

The <u>relationship between the log-normal and the normal distribution</u> becomes apparent through the transformation

$$z = lnx$$

By Ref A-2, we have that if

$$z = g(x)$$

then

$$f_z(z) = \sum f_x(x_i) / |g'(x_i)|$$

where  $x_i$  are all the real roots associated with the value of z and where g'(x) is the derivative of g(x) with respect to x.

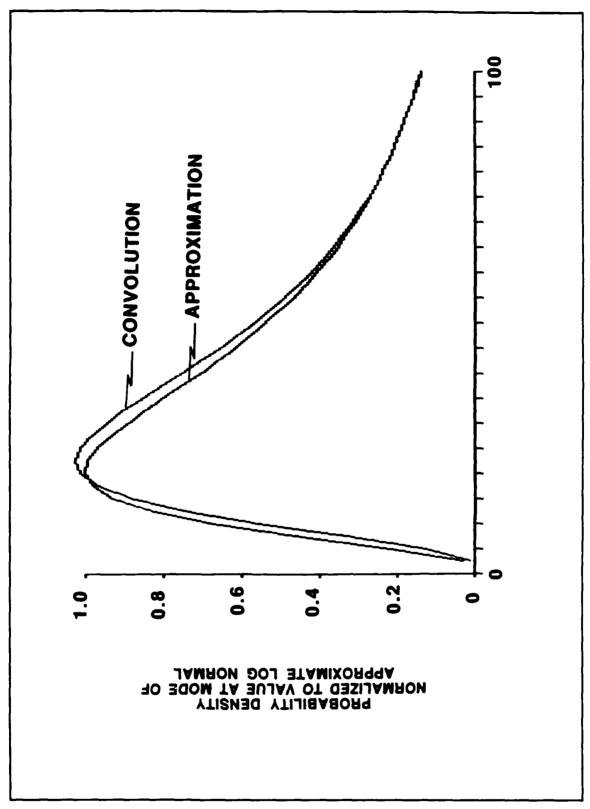


Figure A-2. Comparison of Computer Convolution and Approximation for Thrid Example

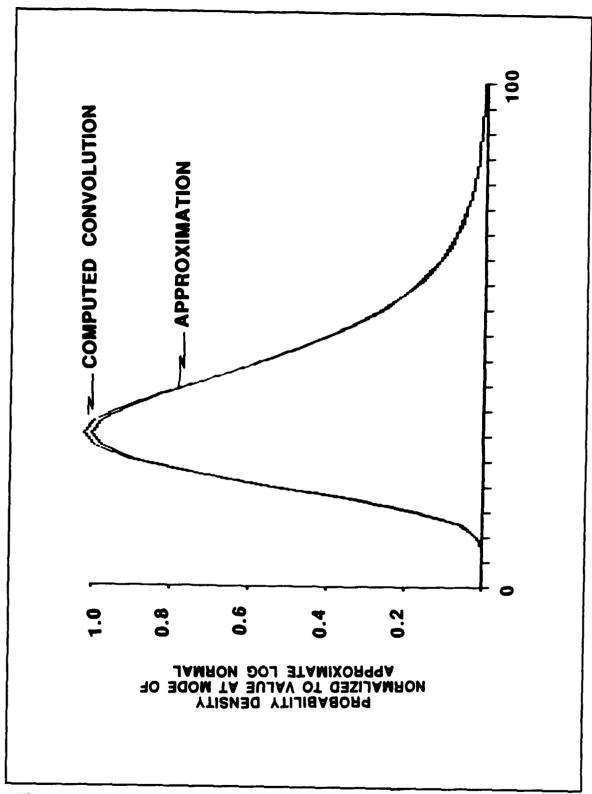


Figure A-3. Comparison of Computer Convolution and Approximation for Second Example

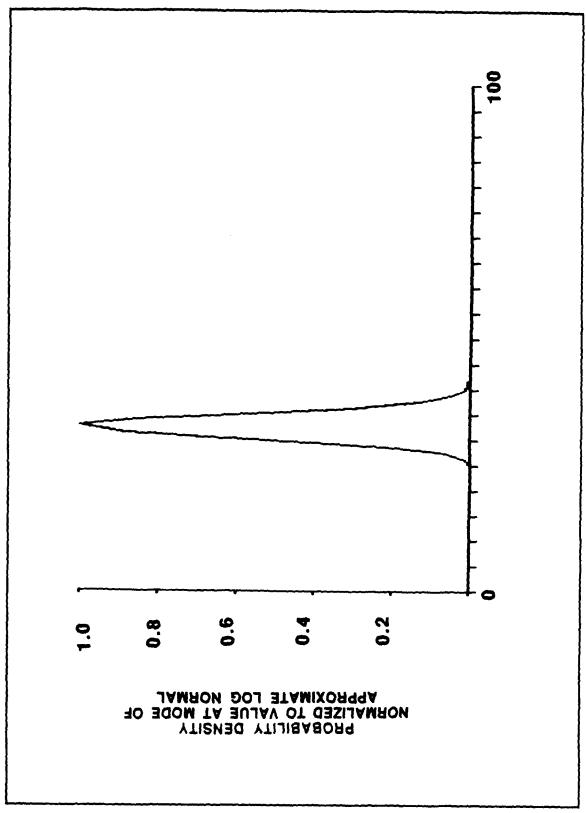


Figure A-4. Comparison of Computer Convolution and Approximation for First Example

In our case, we have only one real root, namely,

$$\exp(z) = x$$

Noting that g'(x) = 1/x and that 1/|g'(x)| is equal to x, we have

$$f_z(z) = x f_x(exp(z))$$

=  $x \exp [-(1/2) (\ln(\exp(z)) - \mu)^2 / \sigma^2] / [\sqrt{(2\pi)} \sigma x]$ 

 $= \exp[-(1/2)(z-\mu)^2/\sigma^2] / \sqrt{(2\pi)}\sigma \qquad -\infty \le z \le \infty$ 

As everyone will recognize, this is the <u>normal distribution function</u>. We also have

$$E(Z) = \mu$$

$$VAR(Z) = \sigma^{2}$$

$$Mode(Z) = \mu$$

$$cv = \sigma/\mu$$

It should be noted that the parameters of the transformed normal density function are readily convertible into the parameters of the log-normal density function; however, the natures of the parameters are very different from one another.

# List of References

- A-1. Parzen, E., Stochastic Processes, Holden-Day, San Fransicso, 1962
- A-2. Papoulis, A., <u>Probability, Random Variables, and Stochastic Processes</u>, McGraw-Hill Book Company, New York, 1965

# Appendix B Curve Fits to Input Data

## Appendix B

### Curve Fits to Input Data

The following tables contain the data used in the calculations given in the report. Table B-1 presents the fitting parameters for the data taken from Figures 1 through 3 of the main body of the report. Table B-2 presents the data from Figure 1, the plot of probability versus system cost for the BSTS, and compares the curve fits for both the log-normal and the normal curve distributions. Table B-3 presents the same data for the SSTS and Table B-4 presents the same data for the SBI.

Table B-1 Curve Fit Parameters

	Log-Norm	al	Norm	ıal
System	μ	σ	μ	σ
BSTS	ln(7.9)	0.12308	7.9	0.97241
SSTS	ln(8.7)	0.18352	8.7	1.5845
SBI	ln(14.33)	0.28108	14.33	3.99300

Table B-2
Curve Fit Data for BSTS

Probability	Log-Normal ty Cost Cost		Normal Cost	
0.00	6.59			
0.10	6.89	6.75	6.60	
0.20	7.16	7.12	<i>7</i> .08	
0.30	<b>7.4</b> 0	7.41	7.39	
0.40	7.64	7.66	7.66	
0.50	<i>7</i> .88	<i>7</i> .90	7.90	
0.60	8.15	8.15	8.14	
0.70	8.42	8.42	8.41	
0.80	8.90	8.76	8.72	
0.90	9.35	9.25	9.14	

Table B-3
Curve Fit Data for SSTS

Probability	Cost	Log-Normal Cost	Normal Cost
0.00	6.00		
0.10	6.84	6.88	6.67
0.20	7.35	7.46	7.37
0.30	7.83	7.91	7.87
0.40	8.22	8.31	8.30
0.50	8.70	8.70	8.70
0.60	9.06	9.11	9.10
0.70	9.50	9.57	9.53
0.80	10.05	10.15	10.03
0.90	10.80	11.00	10.73

Table B-4 Curve Fit Data for SBI

Probability	Cost	Log-Normal Cost	Normal Cost
0.05	9.10	9.03	7.77
0.10	10.00	10.00	9.22
0.20	11.20	11.32	10.98
0.30	12.30	12.38	12.25
0.35	12.95	12.87	12.80
0.40	13.50	13.35	13.33
0.50	14.33	14.33	14.33
0.60	15.50	15.38	15.34
0.70	16.90	16.60	16.42
0.80	18.30	18.14	17.68
0.90	20.20	20.54	19.44
0.95	21.50	22.75	20.90

# Appendix C

# RESULTS FOR LOG-NORMAL DISTRIBUTION FITS FOR DAB ARCHITECTURE ASSUMING INPUT DATA ARE 50% CONFIDENCE VALUES

System	1	CV !	050 (	MILI	SIG	EX	VAR
BSTS	- <del>-</del> -	0.1236	5.40	1.686	0.123	5.441	0.45
SSTS	I I	0.1851	3.80	1.335	0.184	3.865	0.51
SBI	i	0.2867	4.10 !	1.411	0.281	4.265	1.49
GSTS	1	0.1851	1.30 :	0.262	0.184	1.322	0.06
MGBR	i I	0.1236 (	1.30	0.262	0.123	1.310	0.02
ERIS	}	0.1236 }	2.40	0.875	0.123	2.418	0.08
CCSOIF	!	0.2867	4.00 (	1.386	0.281	4.161	1.42
SEI	1	0.1236	3.70 (	1.308	0.123	3.728	0.21
LCH	1	0.1236	3.40 :	1.224	0.123	3.426	0.17
	}	1	;				
	†	\$ 1	i i				
	1	1 1 1	1 1 2				
TOTALS	 		29.40 ¦			29.936	4.44
	!	† 1	!			3.399	
	:	i \$	!			ALPHA	BET

  ACQUISTION 	COSTS FOR	DAB ARCHI	TECTURE		
   System	C50 ;		SIG	EX	VARX
BSTS SSTS SBI GSTS MGBR ERIS CCSOIF SEI LCH	5.40 : 13.60 : 2.00 : 2.00 : 3.51 : 3.30 :	0.956 1.686 2.610 0.693 0.693 1.256 1.194 0.262 1.649	0.123 0.184 0.281 0.184 0.123 0.123 0.281 0.123	2.015 3.537 3.433	0.105 1.033 16.455 0.142 0.062 0.191 0.969 0.026 0.419
TOTALS	38.91	3.678	0.110	39.828 3.685 ALPHA	19.402 BETA

	-	TOT	TALS I				
System	;	EΧ	VARX I				
BSTS	 !	8.061	0.557	PROB	COSTRD	COSTINV	costro
SSTS	}	9.356	1.545	0.1	27.29	34.38	63.6
SBI	!	18.413	17.951	0.2	28.15	36.09	65.6
GSTS	1	3,356	0.202 (	0.3	28.79	37.37	67.1
MGBR	ì	3.325	0.088 :	0.4	29.34	38.50	68.3
ERIS	i	5.955	0.280 :	0.5	29.86	39.59	69.5
COSOIF	i	7.594	2.392	0.6	30.39	40.70	70.8
SEI	!	5.038	0.238	0.7	30.98	41.93	72.1
LCH	1	8.665	0.598 :	0.8	31.68	43.42	73.8
	;		1	0.9	32.68	45.59	76.1
	;		<b>;</b>				
	-		i !				
	! f		1				
	- ! -						
TOTALS	1	69.764	23.851				
	ł	4.245	1				
	1		1				
	1	MU	4.2427 1				
	i 1	SIG	0.0699 :				

				F(8:1)	er 1996 war war tret est toe en 1986 was	
System (	CV !	C50 (	MT	SIG	EΧ	VAR)
BSTS (	0.1236	5.40	1.686	0.123	5.441	0.452
SSTS	0.1851	3.80 :	1.335	0.184	3.865	0.512
SBI :	0.2867	4.10	1.411	0.281	4.265	1.496
GSTS :	0.1851 :	1.30 }	0.262	0.184	1.322	0.060
MGBR (	0.1236 :	1.30 (	0.262	0.123	1.310	0.026
ERIS !	0.1236	2.40 1	0.875	0.123	2.418	0.089
TGBR (	0.1236	1.10	0.095	0.123	1.108	0.019
HEDI (	0.1236 }	1.50 }	0.405	0.123	1.511	0.035
CCSOIF !	0.2867 (	4.25	1.447	0.281	4.421	1.607
SEI !	0.1236	4.11 !	1.413	0.123	4.141	0.262
LCH (	0.1236 (	3.40 (	1.224	0.123	3.426	0.179
1 1	i !	; ;				
TOTALS 1		32.66 (			33.229	4.736
1		1			3.503	
;		!			ALF'HA	BET

	-			INV		
System	1	C50 (	MLI	SIG	EΧ	VARX
	-					
BSTS	;	2.60	0.956	0.123	2.620	0.105
SSTS	;	5.40	1.686	0.184	5.492	1.033
SBI	1	13.60	2.610	0.281	14.148	16.455
GSTS	1	2.00	0.693	0.184	2.034	0.142
MGBR	ť	2.00 ;	0.693	0.123	2.015	0.062
ERIS	1 !	3.51	1.256	0.123	3.537	0.191
TGBR	;	0.00 ;	0.000	0.000	0.000	0.000
HEDI	1	0.00 ;	0.000	0.000	0.000	0.000
COSOIF	1	3.32 (	1.200	0.281	3.454	0.981
SEI	1	1.30 ;	0.262	0.123	1.310	0.026
£€H.	1	5.20 :	1.649	0.123	5.240	0.415

1	TO	TALS				
System	EX	VARX				
BSTS !	8.061	0.557	PROB	COSTRD	COSTINV	COSTTO
SSTS :	9.356	1.545	0.1	30.49	34.39	66.9
SBI	18.413	17.951	0.2	31.39	36.11	68.90
GSTS :	3.356	0.202	0.3	32.05	37.39	70.4
MGBR :	3.325	0.088 :	0.4	32.62	38.52	71.6
ERIS (	5.955	0.280 :	0.5	33.16	39.61	72.9
TGBR 1	1.108	0.019	0.6	33.71	40.72	74.15
HEDI :	1.511	0.035	0.7	34.31	41.95	75.5
COSOIF :	7.875	2.588 /	0.8	35.03	43.45	77.14
SEI :	5.451	0.288	0.9	36.05	45.61	79.46
LCH (	8.665	0.598				
1		1 1				
1		1				
TOTALS (	73.078	24.149				
runneu i	4.292	—————————————————————————————————————				
f f	<b>→</b> 1 = 2 = 2 = 4	;				
1	MU	4.2893				
	SIG	0.0672 (				

# Appendix D

# RESULTS FOR LOG-NORMAL DISTRIBUTION FITS FOR DAB ARCHITECTURE ASSUMING INPUT DATA ARE EXPECTED VALUES

CQUISTION	COSTS FOR	R DAB ARCHIT	CECTURE			
System	CV	E (X)	MU	R&D SIG	EX	VARX
BSTS   SSTS   SBI   GSTS   MGBR   ERIS   CCSOIF   SEI   LCH	0.1236   0.1851   0.2867   0.1236   0.1	5.40   3.80   4.10   1.30   1.30   2.40   4.00   3.70   3.40	1.679 1.318 1.370 0.245 0.255 0.868 1.345 1.301 1.216	0.123 0.184 0.281 0.184 0.123 0.123 0.281 0.123	5.400 3.799 4.093 1.300 1.300 2.400 3.994 3.700 3.400	0.445 0.494 1.378 0.058 0.026 0.088 1.311 0.209 0.176
TOTALS       	 ! !	29.40			29.385 3.380 ALPHA	4.185 BETA
! !	! !	 	3.378	0.070		

ACQUISTION	COSTS FOR	DAB ARCHI	TECTURE		   
	EX	MU	INV SIG	EX	VARX
BSTS   SSTS   SBI   GSTS   MGBR   ERIS   CCSOIF   SEI   LCH	2.60   5.40   13.60   2.00   2.00   3.51   3.30   1.30   5.20	0.948 1.670 2.571 0.676 0.686 1.248 1.154 0.255 1.641	0.123 0.184 0.281 0.184 0.123 0.123 0.281 0.123	2.600 5.400 13.600 2.000 2.000 3.510 3.300 1.300 5.200	0.103 0.999 15.205 0.137 0.061 0.188 0.895 0.026 0.413
   TOTALS   	38.91			38.910 3.661 ALPHA	18.027 BETA
 	 	3.655	0.109		

ACQUISTION	COSTS FOR		IITECTURE			
BSTS SSTS SBI GSTS MGBR ERIS CCSOIF SEI LCH	8.000 9.199 17.693 3.300 3.300 5.910 7.294 5.000 8.600	0.548   1.493   16.583   0.195   0.087   0.276   2.206   0.235   0.589	PROB 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9	COSTRD 26.82 27.65 28.27 28.81 29.31 29.83 30.40 31.07 32.04	COSTINV 33.65 35.31 36.55 37.64 38.68 39.75 40.94 42.38 44.46	65.73 66.96 68.13 69.32 70.63
TOTALS	68.295 4.224	22.212				
	   MU   SIG	4.2215 0.0689	 			

  PROGRAM COS	STS FOR DA	AB ARCHITEC	TURE			   
	CV	EX	MU	R&D SIG	EX	VARX
BSTS   SSTS   SSTS   SSTS   SSTS   SSTS   SSTS   MGBR   ERIS   TGBR   HEDI   CCSOIF   SEI   LCH	0.1236   0.1851   0.2867   0.1851   0.1236   0.1236   0.1236   0.1236   0.1236   0.1236   0.1236	5.40   3.80   4.10   1.30   1.30   2.40   1.10   1.50   4.25   4.11   3.40	1.679 1.318 1.371 0.246 0.255 0.868 0.088 0.398 1.407 1.406 1.216	0.123 0.184 0.281 0.184 0.123 0.123 0.123 0.123 0.281 0.123 0.123	5.400 3.800 4.100 1.300 1.300 2.400 1.100 1.500 4.250 4.110 3.400	0.445   0.495   1.382   0.058   0.026   0.088   0.018   0.034   1.485   0.258   0.176
TOTALS		32.66			32.660 3.486	4.465
		   	3.484	0.065	ALPHA	BETA   

PROGRAM COPROGRAM COSTS FOR DAB ARCHITECTURE								
System	EX	MU	INV SIG	EX	VARX			
BSTS   SSTS   SBI   GSTS   MGBR   ERIS   TGBR   HEDI   CCSOIF   SEI   LCH	2.60   5.40   13.60   2.00   1.80   3.51   0.00   0.00   3.32   1.30   5.20	0.948 1.670 2.571 0.676 0.580 1.248 0.000 0.000 1.160 0.255 1.641	0.123 0.184 0.281 0.184 0.123 0.000 0.000 0.281 0.123 0.123	2.600 5.400 13.600 2.000 1.800 3.510 0.000 0.000 3.320 1.300 5.200	0.103 0.999 15.205 0.137 0.049 0.188 0.000 0.000 0.906 0.026 0.413			
TOTALS	38.73			38.730 3.657 ALPHA	18.027 BETA			
 	i	3.651	0.109					

PROGRAM CO	STS FOR DA	AB ARCHITI	ECTURE			
System	TO	TALS VARX	<b> </b> 			
BSTS   SSTS   SSTS   GSTS   MGBR   ERIS   TGBR   HEDI   CCSOIF   SEI   LCH	8.000 9.200 17.700 3.300 3.100 5.910 1.100 1.500 7.570 5.410 8.600	0.548 1.494 16.587 0.195 0.075 0.276 0.018 0.034 2.391 0.284 0.589	PROB   0.1   0.2   0.3   0.4   0.5   0.6   0.7   0.8   0.9	COSTRD 30.00 30.87 31.51 32.07 32.59 33.13 33.71 34.41 35.40	COSTINV 33.47 35.13 36.37 37.46 38.50 39.57 40.76 42.20 44.28	COSTTOT 65.43 67.38 68.81 70.05 71.23 72.43 73.74 75.31 77.55
TOTALS	71.390	22.492	 			
	MU   SIG	4.2660 0.0664	'   			

# Appendix E

#### RESULTS FOR NORMAL DISTRIBUTION FITS FOR DAB ARCHITECTURE ASSUMING INPUT DATA ARE EXPECTED VALUES

ACQUISTION COSTS FOR DAB ARCHITECTURE							
System	CV	C50	EX	VARX			
BSTS   SSTS   SBI   GSTS   MGBR   ERIS   CCSOIF   SEI   LCH	0.1231   0.1821   0.2786   0.1821   0.1231   0.1231   0.2786   0.1231   0.1231	5.40 3.80 4.10 1.30 1.30 2.40 4.00 3.70 3.40	5.400 3.800 4.100 1.300 1.300 2.400 4.000 3.700 3.400	0.442 0.479 1.305 0.056 0.026 0.087 1.242 0.207 0.175			
TOTALS           	       	29.40	   29.400     	2.005			

COSTS FOR	R DAB ARCH	ITECTURE
C50	EX	VARX
2.60   5.40   13.60   2.00   3.51   3.30   1.30   5.20	2.600 5.400 13.600 2.000 2.000 3.510 3.300 1.300 5.200	0.102 0.967 14.356 0.133 0.061 0.187 0.845 0.026 0.410
38.91	38.910	4.134
	C50   2.60   5.40   13.60   2.00   2.00   3.51   3.30   1.30   5.20	2.60   2.600 5.40   5.400 13.60   13.600 2.00   2.000 2.00   2.000 3.51   3.510 3.30   3.300 1.30   1.300 5.20   5.200

ACQUISTION	COSTS FO	R DAB ARC	HITECTURE			
System	TO	TALS VARX	 			
BSTS   SSTS   SBI   GSTS   MGBR   ERIS   CCSOIF   SEI   LCH	8.000 9.200 17.700 3.300 3.300 5.910 7.300 5.000 8.600	0.544 1.446 15.661 0.189 0.086 0.274 2.087 0.233 0.585	PROB   0.1   0.2   0.3   0.4   0.5   0.6   0.7   0.8   0.9	COSTRD 26.83 27.72 28.35 28.90 29.40 29.90 30.45 31.08 31.97	COSTINV 33.62 35.44 36.75 37.87 38.91 39.95 41.07 42.38 44.20	65.93 67.10 68.33 69.40 70.73
TOTALS	68.310	4.594	   			
†   	MU SIG	68.3100 4.5940	   			

PROGRAM CO	STS FOR DA	AB ARCHITEC	CTURE	   
System	CV	C50	EX	VARX
BSTS   SSTS   SBI   GSTS   MGBR   ERIS   TGBR   HEDI   CCSOIF   SEI   LCH	0.1231 0.1821 0.2786 0.1821 0.1231 0.1231 0.1231 0.1231 0.2786 0.1231 0.1231	5.40 3.80 4.10 1.30 1.30 1.30 2.40 1.10 1.50 4.25 4.11 3.40	5.400 3.800 4.100 1.300 1.300 2.400 1.100 1.500 4.250 4.110 3.400	0.442   0.479   1.305   0.056   0.026   0.087   0.018   0.034   1.402   0.256   0.175
   TOTALS   		32.66	   32.660 	2.069
			 	1

  PROGRAM CO	PROGRAM CO	STS FOR DA	B ARCHIT
System	C50	EX	VARX
BSTS   SSTS   SBI   GSTS   MGBR   ERIS   TGBR   HEDI   CCSOIF   SEI   LCH	2.60 5.40 13.60 2.00 2.00 3.51 0.00 0.00 3.32 1.30 5.20	2.600 5.400 13.600 2.000 2.000 3.510 0.000 0.000 3.320 1.300 5.200	0.102 0.967 14.356 0.133 0.061 0.187 0.000 0.000 0.856 0.026 0.410
TOTALS  I I I	38.93       	38.930     	4.135

PROGRAM CO	OSTS FOR D	AB ARCHIT	ECTURE			
	ТО	TALS	1			
System	EX	VARX	  -			
BSTS SSTS SBI	8.000 9.200 17.700	0.544 1.446 15.661	PROB 0.1 0.2	COSTRD 30.01 30.92	COSTINV 33.64 35.46	COSTTO 65.67 67.73
GSTS MGBR	3.300 3.300	0.189 0.086	0.3 0.4	31.58 32.14	36.77 37.89	69.18 70.43
ERIS TGBR	5.910 1.100	0.274 0.018	0.5 0.6	32.66 33.18	38.93	71.59
HEDI	1.500	0.034	0.7	33.74	41.09	74.00
CCSOIF SEI LCH	7.570 5.410 8.600	2.258 0.282 0.585	0.8	34.40 35.31	42.40 44.22	75.47 77.5
LCR	8.000   	0.363	 			
TOTALS	71.590	4.623	! !			
	   MU   SIG	71.5900 4.6235	!   			

# Appendix F

### SENSITIVITY TO RISK SPLIT BETWEEN PHASES FOR DAB ARCHITECTURE FOR NORMAL DISTRIBUTION FITS

ACQUISTION COSTS FOR DAB ARCHITECTURE								
System	!	CV	050	E/	VARX			
BSTS SSTS SBI GSTS MGBR ERIS CCSOIF SEI LCH		0.1231   0.1821   0.2786   0.1821   0.1231   0.1	5.40 3.80 4.10 1.30 2.40 4.00 3.70 3.40	5.400 3.800 4.100 1.300 2.400 4.000 3.700 3.400	0.945 2.292 10.941 0.286 0.131 0.428 3.845 0.374			
TOTALS	1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1	; ; ;	29.40	   29.400   	4.487			

(ACQUISTIC	N COSTS F	OR DAB ARCH	HITECTURE
System	( 050	EX	VARX (
BSTS SSTS SSTS SSTS MGBR ERIS CCSOIF SEI LCH	5.40 13.60	2,000 2,000 3,510 3,300 1,300	0.024 0.514 13.376 0.075 0.034 0.102 0.291 0.005 0.231
rotals	38.91	38.910	3.828

	77 .77	TALS 1				
System	EX	VARX				
BSTS :	3.000	0.970	PROB	COSTRD	COSTINV	COSTTOT
SSTS :	9.200	2.807 :	0.1	23.66	34.01	60.76
SBI :	17.700	24.317 (	0.2	25,64	35.70	63.36
GSTS :	3.300	0.361	0.3	27.06	36.91	65,23
MGBR (	3,300	0.165	0.4	28.27	37.95	66.83
ERIS :	5.910	0.529	0.5	29.40	38.91	68.31
CCSOIF :	7.300	4.136	0.6	30.53	39.87	69.79
SEI	5.000	0.379	0.7	31.74	40.91	71.39
LCH :	8.600	1.121	0.8	33.16	42.12	73.26
		!	0.9	35.14	43.81	75.86
		:	2.4.2			
i						
TOTALS :	68.310	5.898				
		!				
:	MU	68.3100				
•	516	5.8979				

System	:	CV	C50 :	EX	VARX
BSTS	}	0.1231	5.40	5.400	0.945
SSTS	!	0.1821 :	3.80	3.800	2,292
SBI	í	0.2786	4.10	4.100	10.941
GSTS	;	0.1821	1.30	1.300	0.286
MGBR	;	0.1231 /	1.30	1.300	0.131
ERIS	!	0.1231	2.40	2.400	0.428
TGBR	!	0.1231	1.10 (	1.100	0.131
HEDI	;	0.1231	1.50 (	1.500	0.428
COSDIF	!	0.2786	4.25	4.250	4.165
SEI	i	0.1231	4.11	4.110	0.439
LCH	1	0.1231	3.40	3.400	0.890
TOTAL5	<u></u>		32.66	32.660	4.591

PROGRAM C	OSTS FOR DAI	B ARCHITE	CTURE
System	C50 :	EX	VARX
BSTS SSTS SBI GSTS MGBR ERIS TGBR HEDI CCSOIF SEI LCH	2.60   5.40   13.60   2.00   2.00   3.51   0.00   0.00   13.32   1.30   5.20   1	2.600 5.400 13.600 2.000 2.000 3.510 0.000 0.000 3.320 1.300 5.200	0.024 0.514 13.376 0.075 0.034 0.102 0.000 0.000 0.282 0.005 0.231
TOTALS	38.93	38.930	3.827

	ROGRAM C	OST	5 FOR D	AB ARCHIT	EC	TURE	e and one are as a control on the			
!										1
		•		TALS	•					
i	System	;	EX	VARX	ř 3					i
!	BSTS	– –	 -8.000	0.970	' i !	FROB	COSTRD	COSTINV	costror	:
į			- <del>-</del>	2.807		0.1	26.78	34.03		
•				24.317			28.81			
1		-		0.361	!	0.3	30.27	36.93	68.47	1
1	<u>-</u>	-		0.165	!		31.51	37.97	70.09	:
1		1	5.910	0.529	•	0.5	32.66	38.93	71.59	•
1	TGBR	1	1.100		!		33.81	39.89	,	1
1	HEDI	1	1.500	0.131	•	0.0	35.05	40.93		
		1								:
i	COSOIF	•	7.570	4.448	•		36.51			,
i			5.410	0.444	i	0.9	38.54	43.83	79.24	:
i	LCH	i	8.600	1.121	i					•
1		i			1					i
!		1			1 1					i
					· †					!
;	TOTALS	1	71.590	5.977	}					:
1		ì			1					!
i		1			į					i I
:		!	MLI	71.5900	!					1
1		j 1	SIG	5.9766	ļ					i

# Appendix G

### RESULTS FOR CORRELATED NORMAL DISTRIBUTION FITS FOR ALL ARCHITECTURES

  ACQUISITI	ON COSTS FO	OR DAB ARCH	HITECTURE	   
	l CV	   C50	R&D EX	VARX
BSTS SSTS SSTS SBI GSTS MGBR ERIS CCSOIF SEI LCH	0.1231   0.1821   0.2786   0.1821   0.1231   0.1231   0.2786   0.1231   0.1231	5.40 3.80 4.10 1.30 1.30 2.40 4.00 3.70 3.40	5.400 3.800 4.100 1.300 1.300 2.400 4.000 3.700 3.400	0.442   0.479   1.305   0.056   0.026   0.087   1.242   0.207   0.175
TOTALS	         	29.40     	29.170 MUX3D	2.005   SIGR&D   

ACQUISITION	COSTS FO	OR DAB ARCI	  HITECTURE
System	C50	EX	   VARX
BSTS     SSTS     SBI     GSTS     MGBR     ERIS     CCSOIF     SEI     LCH	2.60 5.40 13.60 2.00 2.00 3.51 3.30 1.30 5.20	2.600 5.400 13.600 2.000 2.000 3.510 3.300 1.300 5.200	0.102   0.967   14.356   0.133   0.061   0.187   0.845   0.026   0.410
 	38.91	     38.910   MUINV 	4.134   SIGINV

  ACQUISITIO	ON COSTS F	OR DAB AR	CHITECTURE			
      System	(TO	TALS VARX	Correlatio	on betwee	n Phases	0.50
BSTS SSTS SSTS SSTS SSTS SSTS SSTS SSTS	8.000   9.200   17.700   3.300   3.300   5.910   7.300   5.000   8.600	0.757 2.126 19.989 0.275 0.126 0.402 3.112 0.306 0.853	PROB 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9	COSTRD 26.83 27.72 28.35 28.90 29.40 29.90 30.45 31.08 31.97	COSTINV 33.62 35.44 36.75 37.87 38.91 39.95 41.07 42.38 44.20	COSTTOT 57.39 61.15 63.86 66.17 68.31 70.45 72.76 75.47 79.23
TOTALS	   68.310   MU	5.286 SIG	·			
 	MU SIG	68.3100 8.5294				

PROGRAM COSTS FOR DAB ARCHITECTURE						
System	CV	C50 (	EX	VARX		
BSTS   SSTS   SBI   GSTS   MGBR   ERIS   TGBR   HEDI   CCSOIF   SEI   LCH	0.1231   0.1821   0.2786   0.1821   0.1231   0.1231   0.1231   0.2786   0.1231   0.1231	5.40   3.80   4.10   1.30   2.40   1.10   1.50   4.25   4.11   3.40	5.400 3.800 4.100 1.300 1.300 2.400 1.100 1.500 4.250 4.110 3.400	0.442 0.479 1.305 0.056 0.026 0.087 0.018 0.034 1.402 0.256 0.175		
TOTALS		32.66            	32.660 MUR&D	2.069 SIGR&D		

PROGRAM COSTS FOR DAB ARCHITECTURE						
System	C50	EX	VARX			
BSTS SSTS SSTS SSTS SSTS SSTS SSTS SSTS	2.60 5.40 13.60 2.00 2.00 3.51 0.00 0.00 3.32 1.30 5.20	2.600 5.400 13.600 2.000 2.000 3.510 0.000 0.000 3.320 1.300 5.200	0.102   0.967   14.356   0.133   0.061   0.187   0.000   0.000   0.856   0.026   0.410			
TOTALS	38.93	38.930   MUINV	4.135 SIGINV			
   	   	'   				

PROGRAM CO	OSTS FOR D	AB ARCHIT	ECTURE			
System	EX	VARX	1			
BSTS SSTS SBI GSTS MGBR ERIS TGBR HEDI CCSOIF SEI LCH	8.000 9.200 17.700 3.300 3.300 5.910 1.100 1.500 7.570 5.410 8.600	0.757 2.126 19.989 0.275 0.126 0.402 0.018 0.034 3.353 0.363 0.853	PROD  1 0.1  0.1  0.1  0.1  0.1  0.1  0.1  1 0.1  1 0.1  1 0.1	1 30.01 30.92 31.58 4 32.14 5 32.66 6 33.18 7 33.74 8 34.40	COSTINV 33.64 35.46 36.77 37.89 38.93 39.97 41.09 42.40 44.22	COSTTOT 59.72 63.81 66.75 69.26 71.59 73.92 76.43 79.37 83.46
TOTALS	     71.590   MU	5.319 SIG	 			
	   MU   SIG	71.5900 9.2740	l   			

#### Correlation

Between	Acquis	sition Co	sts	Pr	ogram Cos	ts
Programs	20%	50%	80%	20%	50%	80%
0.0	63.87	68.31	72.75	67.13	71.59	76.05
0.1	63.21	68.31	73.41	66.29	71.59	76.89
0.2	62.63	68.31	73.99	65.58	71.59	77.60
0.3	62.10	68.31	74.52	64.93	71.59	78.25
0.4	61.61	68.31	75.01	64.35	71.59	78.83
0.5	61.15	68.31	75.47	63.81	71.59	79.37
0.6	60.73	68.31	75.89	63.30	71.59	79.88
0.7	60.32	68.31	76.30	62.83	71.59	80.35
0.8	59.93	68.31	76.69	62.37	71.59	80.81
0.9	59.57	68.31	77.05	61.94	71.59	81.24
1.0	59.21	68.31	77.41	61.53	71.59	81.65

ACQUISITION COSTS FOR ERIS ARCHITECTURE						
System	CV	C50	EX	VARX		
BSTS   SSTS   GSTS   MGBR   ERIS   CCSOIF   SEI   LCH	0.1231 0.1821 0.1821 0.1231 0.1231 0.2786 0.1231 0.1231	5.40 3.80 1.30 1.30 2.40 3.98 2.81 1.30	5.400 3.800 1.300 1.300 2.400 3.980 2.810 1.300	0.442   0.479   0.056   0.026   0.087   1.229   0.120   0.026		
TOTALS		22.29       	   22.290   MUR&D 	1.570   SIGR&D		

  ACQUISITION 	COSTS FO	R ERIS ARC	  HITECTUR: 
	C50	INV	
System		EX	VARX
BSTS   SSTS   SSTS   GSTS   MGBR   ERIS   CCSOIF   SEI   LCH	2.60	2.600	0.102
	4.32	4.320	0.619
	2.75	2.750	0.251
	2.00	2.000	0.061
	6.02	6.020	0.549
	3.90	3.900	1.181
	0.82	0.820	0.010
	1.60	1.600	0.039
	24.01	24.010 MUINV	1.677   SIGI <b>NV</b>

ACQUISITIO	ON COSTS F	OR ERIS A	RCHITECTUR	 E		
      System	TO   EX	TALS VARX	Correlati	on betwee	n Phases	0.50
BSTS SSTS SSTS SSTS SSTS SSTS SSTS SSTS		0.757 1.642 0.425 0.126 0.855 3.615 0.165 0.096	PROB   0.1   0.2   0.3   0.4   0.5   0.6   0.7   0.8   0.9	COSTRD 20.28 20.97 21.47 21.90 22.29 22.68 23.11 23.61 24.30		43.68 45.04
TOTALS	46.300 MU	2.771 SIG	1			
!   	   MU   SIG	46.3000 5.0295	   			

PROGRAM COSTS FOR ERIS ARCHITECTURE						
System	CV	C50	EX	VARX		
BSTS   SSTS   SSTS   GSTS   MGBR   ERIS   TGBR   HEDI   CCSOIF   SEI   LCH	0.1231   0.1821   0.2786   0.1231   0.1	5.40 3.80 4.10 1.30 1.30 2.40 1.10 1.50 4.25 4.11 3.40	5.400 3.800 4.100 1.300 1.300 2.400 1.100 1.500 4.250 4.110 3.400	0.442   0.479   1.305   0.056   0.026   0.087   0.018   0.034   1.402   0.256   0.175		
   TOTALS   		32.66	   32.660   MUR&D	2.069 SIGR&D		

PROGRAM COSTS FOR ERIS ARCHITECTURE							
System	C50	EX	VARX				
BSTS SSTS SBI GSTS MGBR ERIS TGBR HEDI CCSOIF SEI LCH	2.60 4.32 0.00 2.75 2.00 6.02 0.00 0.00 3.90 0.82 1 0	2.600 4.320 0.000 2.750 2.000 6.020 0.000 0.000 3.900 0.820 1.600	0.102   0.619   0.000   0.251   0.061   0.549   0.000   0.000   1.181   0.010   0.039				
TOTALS	24.01	24.010   MUINV	1.677   SIGINV   				

PROGRAM CO	STS FOR E	RIS ARCHI	TECTURE				
 	ТО	TALS	1				1
System	EX	VARX	[				1
BSTS	8.000	0.757	PROB	COSTRD	COSTINV	COSTTOT	1
SSTS	8.120	1.642	0.1	30.01	21.86	48.38	l
SBI	4.100	1.305	0.2	30.92	22.60	51.23	l
GSTS	4.050	0.425	0.3	31.58	23.14	53.29	ļ
MGBR	3.300	0.126	0.4	32.14	23.59	55.04	ĺ
ERIS	8.420	0.855	0.5	32.66	24.01	56.67	1
TGBR	1.100	0.018	0.6	33.18	24.43	58.30	ĺ
HEDI	1.500	0.034	0.7	33.74	24.88	60.05	ĺ
CCSOIF	8.150	3.869	0.8	34.40	25.42	62.11	1
! SEI	4.930	0.317	0.9	35.31	26.16	64.96	ĺ
LCH	5.000	0.296	1				ı
			1				ĺ
			į .				i
			!				i
TOTALS	56.670	3.106					j
ļ	MU	SIG					ĺ
							ĺ
<b>l</b>	MU	56.6700					ĺ
l i	SIG	6.4788	I				1

Correlation Between Phases 0.50

Correlat: Between		sition Co	sts	Pr	ogram Cos	ts
Programs	20%	50%	80%	20%	้50%	80%
0.0	43.97	46.3	48.63	54.06	56.67	59.28
0.1	43.49	46.3	49.11	53.30	56.67	60.04
0.2	43.08	46.3	49.52	52.68	56.67	60.66
0.3	42.72	46.3	49.88	52.15	56.67	61.19
0.4	42.38	46.3	50.22	51.67	56.67	61.67
0.5	42.08	46.3	50.52	51.23	56.67	62.11
0.6	41.80	46.3	50.80	50.83	56.67	62.51
0.7	41.53	46.3	51.07	50.45	56.67	62.89
0.8	41.27	46.3	51.33	50.10	56.67	63.24
0.9	41.03	46.3	51.57	49.76	56.67	63.58
1.0	40.80	46.3	51.80	49.44	56.67	63.90

ACQUISITION	COSTS FO	R ERIS/HEDI		TURE
System	CV	C50	EX	VARX
BSTS   SSTS   GSTS   MGBR   ERIS   TGBR   HEDI   CCSOIF   SEI   LCH	0.1231   0.1821   0.1821   0.1231   0.1231   0.1231   0.1231   0.2786   0.1231   0.1231	5.40   3.80   1.30   1.30   2.40   1.10   1.50   4.23   3.22   1.30	5.400 3.800 1.300 1.300 2.400 1.100 1.500 4.230 3.220 1.300	0.442   0.479   0.056   0.026   0.087   0.018   0.034   1.389   0.157   0.026
TOTALS	         	25.55	25.550 MUR&D	1.647   SIGR&D

ACQUISITION	COSTS FO	R ERIS/HEDI	ARCHIT
   System	C50	EX	 VARX
BSTS   SSTS   GSTS   MGBR   ERIS   TGBR   HEDI   CCSOIF   SEI   LCH	2.60   4.32   2.75   2.00   5.08   6.16   5.86   5.93   1.26   1.60	2.600 4.320 2.750 2.000 5.080 6.160 5.860 5.930 1.260 1.600	0.102 0.619 0.251 0.061 0.391 0.575 0.520 2.729 0.024 0.039
!	37.56      	37.560 MUINV	2.305 SIGINV

ACQUISITION	ON COSTS F	OR ERIS/H	EDI ARCHITI	ECTURE		 
     System	TO	TALS VARX	Correlation	on betwee	n Phases	0.50
BSTS SSTS SSTS SSTS SSTS SSTS SSTS SSTS	8.000 8.120 4.050 3.300 7.480 7.260 7.360 10.160 4.480 2.900	0.757 1.642 0.425 0.126 0.663 0.696 0.688 6.065 0.243 0.096	PROB   0.1   0.2   0.3   0.4   0.5   0.6   0.7   0.8   0.9	COSTRD 23.44 24.17 24.69 25.14 25.55 25.96 26.41 26.93 27.66	36.36 36.98	60.11   61.67   63.11   64.55
   TOTALS     	63.110   MU   MU   SIG	3.376 SIG 63.1100 5.7460	: (   			

PROGRAM CO			R&D	
System	CA i	C50	EX	VARX
BSTS   SSTS   SSTS   SBI   GSTS   MGBR   ERIS   TGBR   HEDI   CCSOIF   SEI   LCH	0.1231   0.1821   0.2786   0.1821   0.1231   0.1231   0.1231   0.1231   0.1231   0.1231   0.1231   0.1231	5.40   3.80   4.10   1.30   2.40   1.10   1.50   4.25   4.11   3.40	5.400 3.800 4.100 1.300 1.300 2.400 1.100 1.500 4.250 4.110 3.400	0.442 0.479 1.305 0.056 0.026 0.087 0.018 0.034 1.402 0.256 0.175
TOTALS           		32.66	32.660 MUR&D	2.069 SIGR&D

PROGRAM CO	OSTS FOR E	RIS/HEDI AF	  CHITECTU 
System	   C50	INV   EX	VARX
BSTS SSTS SSTS SSTS SSTS SSTS SSTS SSTS	2.60 4.32 0.00 2.75 2.00 5.08 6.16 5.86 5.93 1.26 1.60	2.600 4.320 0.000 2.750 2.000 5.080 6.160 5.860 5.930 1.260 1.600	0.102   0.619   0.000   0.251   0.061   0.391   0.575   0.520   2.729   0.024   0.039
TOTALS	   37.56       	37.560   MUINV 	2.305   SIGINV   

PROGRAM CO	STS FOR E	 RIS/HEDI	ARCHITECTU	 RE		
System	TO	TALS VARX	1			
BSTS   SSTS   SBI   GSTS   MGBR   ERIS   TGBR   HEDI   CCSOIF   SEI   LCH	8.000 8.120 4.100 4.050 3.300 7.480 7.260 7.360 10.180 5.370 5.000	0.757 1.642 1.305 0.425 0.126 0.663 0.696 0.688 6.088 0.359 0.296	PROB 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9	COSTRD 30.01 30.92 31.58 32.14 32.66 33.18 33.74 34.40 35.31	COSTINV 34.61 35.63 36.36 36.98 37.56 38.14 38.76 39.49 40.51	COSTTC 60.2 63.6 66.1 68.2 70.2 72.1 74.2 76.7 80.1
TOTALS	70.220 MU	3.612 SIG	 			
	MU SIG	70.2200 7.7772	1			

Correlation Between Phases 0.50

Correlatio	n					
Between -	Acqui	sition Co	sts	Pr	ogram Cos	ts
Programs	20%	50%	80%	20%	50%	80%
0.0	60.29	63.11	65.93	67.19	70.22	73.25
0.1	59.79	63.11	66.43	66.24	70.22	74.20
0.2	59.36	63.11	66.86	65.47	70.22	74.97
0.3	58.97	63.11	67.25	64.81	70.22	75.63
0.4	58.62	63.11	67.60	64.23	70.22	76.21
0.5	58.29	63.11	67.93	63.69	70.22	76.75
0.6	57.98	63.11	68.24	63.20	70.22	77.24
0.7	57.69	63.11	68.53	62.74	70.22	77.70
0.8	57.42	63.11	68.80	62.31	70.22	78.13
0.9	57.15	63.11	69.07	61.89	70.22	78.55
1.0	56.90	63.11	69.32	61.50	70.22	78.94

  ACQUISITION 	COSTS FO	R BP ARCHI	TECTURE	   
System	CV	C50	EX	VARX
BSTS   SSTS   SSTS   SBI   CCSOIF   SEI   LCH	0.1231   0.1821   0.2786   0.2786   0.1231   0.1231	5.40   3.80   3.50   3.52   2.83   3.40	5.400 3.800 3.500 3.520 2.830 3.400	0.442   0.479   0.951   0.962   0.121   0.175
	       	22.45	22.450 MUR&D	1.769   SIGR&D     

  ACQUISITION 	COSTS FO	OR BP ARCH	TECTURE
   System	C50	INV   EX	VARX
BSTS   SSTS   SBI   SBI   CCSOIF   SEI   LCH	2.60 4.32 1.10 1.97 0.55 4.91	2.600 4.320 1.100 1.970 0.550 4.910	0.102 0.619 0.094 0.301 0.005 0.365
TOTALS	15.45	15.450   MUINV 	1.219 SIGINV

ACQUISITIO	ON COSTS F	OR BP ARC	CHITECTURE			
System	TO	TALS VARX	Correlatio	on betwee	n Phases	0.50
BSTS SSTS SBI CCSOIF SEI LCH	8.000 8.120 4.600 5.490 3.380 8.310	0.757 1.642 1.344 1.801 0.150 0.793	PROB 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9	COSTRD 20.19 20.97 21.53 22.01 22.45 22.89 23.37 23.93 24.71	COSTINV 13.89 14.43 14.81 15.14 15.45 15.76 16.09 16.47 17.01	COSTTOT 32.06 34.07 35.52 36.75 37.90 39.05 40.28 41.73 43.74
TOTALS	37.900 MU	2.547 SIG	-      -			
	MU SIG	37.9000 4.5637	   			

  PROGRAM CC	STS FOR BP	ARCHITECT	URE	   
   System	CV	C50	EX	VARX
BSTS     SSTS     SBI     GSTS     MGBR     ERIS     TGBR     HEDI     CCSOIF     SEI     LCH	0.1231   0.1821   0.2786   0.1821   0.1231   0.1	5.40 3.80 3.50 1.30 1.30 2.40 1.10 1.50 4.25 4.11 3.40	5.400 3.800 3.500 1.300 1.00 2.400 1.100 1.500 4.250 4.110 3.400	0.442   0.479   0.951   0.056   0.026   0.087   0.018   0.034   1.402   0.256   0.175
TOTALS		32.06	32.060 MUR&D	1.981   SIGR&D

  PROGRAM CO	STS FOR BP	ARCHITECT	 CURE
      System	C50	EX	 VARX
BSTS SSTS SSTS SSTS SSTS SSTS SSTS SSTS	2.60   4.32   1.10   0.00   0.00   0.00   0.00   1.97   0.55   4.91	2.600 4.320 1.100 0.000 0.000 0.000 0.000 1.970 0.550 4.910	0.102 0.619 0.094 0.000 0.000 0.000 0.000 0.301 0.005 0.365
TOTALS	   15.45   	15.450 MUINV	1.219 SIGINV

PROGRAM CO	STS FOR B	P ARCHITEC	CTURE			
   System	TO	TALS VARX				
BSTS   SSTS   SSTS   SSTS   GSTS   MGBR   ERIS   TGBR   HEDI   CCSOIF   SEI   LCH	8.000 8.120 4.600 1.300 1.300 2.400 1.100 1.500 6.220 4.660 8.310	0.757 1.642 1.344 0.056 0.026 0.087 0.018 0.034 2.353 0.295 0.793	PROB 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9	COSTRD 29.52 30.40 31.03 31.56 32.06 32.56 33.09 33.72 34.60	COSTINV 13.89 14.43 14.81 15.14 15.45 15.76 16.09 16.47 17.01	COSTTO 40.4 42.8 44.6 46.1 47.5 48.8 50.3 52.1
TOTALS	47.510 MU MU	2.721 SIG 47.5100 5.5027				

Correlation Between Phases 0.50

Correlati	Lon					
Between	Acquis	sition Co	sts	Pr	ogram Cos	ts
Programs	20%	50%	80%	20%	50%	80%
0.0	35.76	37.9	40.04	45.23	47.51	49.79
0.1	35.33	37.9	40.47	44.61	47.51	50.41
0.2	34.97	37.9	40.83	44.10	47.51	50.92
0.3	34.64	37.9	41.16	43.65	47.51	51.37
0.4	34.34	37.9	41.46	43.26	47.51	51.76
0.5	34.07	37.9	41.73	42.89	47.51	52.13
0.6	33.82	37.9	41.98	42.56	47.51	52.46
0.7	33.58	37.9	42.22	42.24	47.51	52.78
0.8	33.35	37.9	42.45	41.94	47.51	53.08
0.9	33.13	37.9	42.67	41.66	47.51	53.36
1 0	32.92	37.9	42.88	41.39	47.51	53.63

  ACQUISITIO 	N COSTS FO	OR SBI ARCI	HITECTURE	   
System	CV	C50	EX	VARX
BSTS     SSTS     SBI     CCSOIF     SEI     LCH	0.1231 0.1821 0.2786 0.2786 0.1231 0.1231	5.40 3.80 4.10 3.53 2.91 3.40	5.400 3.800 4.100 3.530 2.910 3.400	0.442   0.479   1.305   0.967   0.128   0.175
   TOTALS   		23.14     	23.140 MUR&D	1.870   SIGR&D     

  ACQUISITION	COSTS FO		HITECTURE
System	C50	EX	VARX
BSTS   SSTS   SBI   CCSOIF   SEI   LCH	2.60 5.40 25.96 1.99 1.59 9.59	2.600 5.400 25.960 1.990 1.590 9.590	0.102 0.967 52.308 0.307 0.038 1.394
TOTALS     TOTALS   	47.13	47.130   MUINV 	7.424 SIGINV

ACQUISITIO	ON COSTS F	OR SBI AR	CHITECTURE		**	
System	TO	TALS VARX	Correlatio	on betwee	n Phases	0.50
BSTS SSTS SBI CCSOIF SEI LCH	8.000 9.200 30.060 5.520 4.500 12.990	0.757 2.126 61.875 1.820 0.237 2.063	PROB 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9	COSTRD 20.75 21.57 22.16 22.67 23.14 23.61 24.12 24.71 25.53	COSTINV 37.63 40.90 43.26 45.26 47.13 49.00 51.00 53.36 56.63	COSTTOT 55.95 60.88 64.44 67.46 70.27 73.08 76.10 79.66 84.59
TOTALS	70.270   MU	8.299 SIG	-   			
	   MU   SIG	70.2700 11.1851				

PROGRAM CO			R&D-	
System	cv i	C50	EX	VARX
BSTS   SSTS   SBI   GSTS   MGBR   ERIS   TGBR   HEDI   CCSOIF   SEI   LCH	0.1231   0.1821   0.2786   0.1821   0.1231   0.1231   0.1231   0.1231   0.2786   0.1231   0.1231	5.40 3.80 4.10 1.30 2.40 1.10 1.50 4.25 4.11 3.40	5.400 3.800 4.100 1.300 1.300 2.400 1.100 1.500 4.250 4.110 3.400	0.442 0.479 1.305 0.056 0.026 0.087 0.018 0.034 1.402 0.256 0.175
TOTALS		32.66	   32.660   MUR&D   	2.069 SIGR&D

PROGRAM COSTS FOR SBI ARCHITECTURE										
System	   C50	EX	VARX							
BSTS SSTS SBI GSTS MGBR ERIS TGBR HEDI CCSOIF SEI LCH	2.60   5.40   5.40   25.96   0.00   0.00   0.00   0.00   1.99   1.59   9.59	2.600 5.400 25.960 0.000 0.000 0.000 0.000 0.000 1.990 1.590 9.590	0.102 0.967 52.308 0.000 0.000 0.000 0.000 0.307 0.038 1.394							
   TOTALS     	   47.13         	47.130 MUINV	7.424 SIGINV							

ROGRAM CO	STS FOR S	BI ARCHIT	ECTURE			
 System	TO	TALS VARX				
BSTS   SSTS   SSI   GSTS   MGBR   ERIS   TGBR   HEDI   CCSOIF   SEI   LCH	8.000 9.200 30.060 1.300 2.400 1.100 1.500 6.240 5.700 12.990	0.757 2.126 61.875 0.056 0.026 0.087 0.018 0.034 2.366 0.393 2.063	PROB 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9	COSTRD 30.01 30.92 31.58 32.14 32.66 33.18 33.74 34.40 35.31	COSTINV 37.63 40.90 43.26 45.26 47.13 49.00 51.00 53.36 56.63	COSTTOT 64.40 69.70 73.52 76.77 79.79 82.81 86.06 89.88 95.18
TOTALS	79.790 MU	8.355 SIG	 			
   	MU SIG	79.7900 12.0225	   			

Correlation Between Phases 0.50

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Between	Acquis	ition Co	osts	Pr	ogram Cost	cs
Programs	20%	50%	80%	20%	50%	80%
0.0	63.31	70.27	77.23	72.78	79.79	86.80
0.1	62.76	70.27	77.78	72.07	79.79	87.51
0.2	62.25	70.27	78.29	71.41	79.79	88.17
0.3	61.77	70.27	78.77	70.81	79.79	88.77
0.4	61.32	70.27	79.22	70.24	79.79	89.34
0.5	60.88	70.27	79.66	69.70	79.79	89.88
0.6	60.47	70.27	80.07	69.19	79.79	90.39
0.7	60.08	70.27	80.46	68.71	79.79	90.87
0.8	59.69	70.27	80.85	68.24	79.79	91.34
0.9	59.33	70.27	81.21	67.80	79.79	91.78
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